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General Equilibrium Models of Polycentric Urban Land Use with Endogenous Congestion and Job Agglomeration[1]

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A computable general equilibrium model of urban land use is developed with land allocated to houses, production, and roads. Traffic congestion and employment location are endogenous. Consumers choose job and home locations and want to shop everywhere. Without scale economies in shopping, production is dispersed with rent, wage, commodity price, and net density gradients all peaking at the center of the space. When scale economies in shopping are strong relative to the cost of traffic congestion, dispersion becomes unstable. Multiple equilibria emerge as production agglomerates into a number of centers. Our algorithm tests the stability of equilibria and finds perturbations that set off transitions to other equilibria. The number of centers trades off the benefits from agglomeration against those from access. With stronger agglomeration, there are fewer and bigger centers and utility is higher with fewer centers. With higher congestion, the number of centers increases and utility is higher with more centers. © 1996 Academic Press, Inc.

1. INTRODUCTION

A weakness in urban economic theory is that it has relied too heavily on the monocentric city model.[2] A single job center runs counter to the evidence that has accumulated in the empirical literature on employment

[1] This paper is based in part on Kim [23], Ph.D. dissertation of the second author, completed under the supervision and direction of the first author. The authors thank Richard Arnott, Ping Wang, and Ken Small for their stimulating comments and questions, and also the referees and editor of the journal for comments regarding the exposition.

[2] See Wheaton [39] and Berry and Kim [7] for critiques of the monocentric paradigm.

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subcenters.[3] But, the Achilles' heel of the monocentric model is that it fails to explain that job location—even in a single center—is not exogenous but depends on other determinants of urban form.[4]

The monocentric model has been difficult to jettison because of its simplicity. Would an alternative polycentric model be too complex and intractable? A reasonably tractable polycentric model can be based on the assumption that production and residential uses can occur everywhere in an initially featureless space but become interdependent by the consumption-related travel decisions of consumers and the interindustry linkages among firms. Consumers value access not only to jobs but also to shopping centers and producers value access to other producers, to labor, and to customers. The location of production and, hence, of jobs is endogenous as is the location of residences and, hence, of labor. The model presented here determines conditions under which employment, like residential land use, is dispersed throughout the urban space and a set of conditions under which it clusters into a discrete number of subcenters. The monocentric city arises as the total clustering of jobs.

This paper is related to three separate lines of research in the literature. First are partial equilibrium analyses of cities with two job centers, by White [40], Wieand [41], Sullivan [34], Sivitanidou and Wheaton [30], Helsley and Sullivan [19], and Usowski [35], in which the locations of the two centers are given. In our treatment, there are no prespecified locations or numbers of subcenters, and our model is a fully closed general equilibrium spatial economy.

Second are models which do not prespecify any centers but derive the result that rent gradients and land use densities peak around the most accessible place in the urban space. To our knowledge, the first such models were developed by Karlqvist and Lundqvist [22] and Beckmann [6] who examined interactions among households; and Solow and Vickrey [31],

[3] There are two recent windows into this empirical literature. The first is the July 1991 special issue of *Regional Science and Urban Economics* on the "Causes and Consequences of the Changing Urban Form," which includes articles by Giuliano and Small on Los Angeles [14], Mieszkowski and Smith on Houston [26], Dowall and Treffeisen on Bogota [11], and Shukla and Waddell on Dallas–Fort Worth [29]. See also the prior papers by Peter Gordon and Harry Richardson, e.g., [15] and Gordon *et al.* [16]. The second is the January 1993 special issue of *Geographical Analysis* on "The Multinodal Metropolis" which contains articles by Waddell *et al.* [37], Hoch and Waddell [20], Waddell and Shukla [38], Archer and Smith [5], and Waddell [36].

[4] Lowry [25] recognized that retail employment location directly depends on the distribution of customers but assumed that basic employment was insensitive to the distribution of population. Other authors (see Steinnes and Fisher [33], Steinnes [32], and Boarnet [8]) have tested empirically the idea that employment and residential location are interdependent. Waddell [36] presents estimates of a nested logit model of households' joint workplace and housing choices.

and Borukhov and Hochman [9] who examined contacts among firms. The central peaking derived in these papers follows from the assumption that each unit of economic activity (consumer or a firm) interacts with all others.[5] This drives them to bid up prices and increase densities in the accessible locations. Papageorgiou and Thisse [28] developed a version of these partial equilibrium models with both firms and households. In their model, central peaking occurs by forcing consumers to visit each firm. Fujita and Ogawa [13] introduced the presence of non-pecuniary externalities in production arising from the proximity of firms to one another. They found multiple equilibria. Fujita [12] also showed that purely pecuniary interactions between consumers and firms also lead to non-monocentric patterns under monopolistic competition.[6]

Third are articles in the literature which explain the formation of agglomerations endogenously as a tug-of-war between positive externalities from the co-location of economic agents and the costs of interaction among agents at different locations.[7] Papageorgiou and Smith [27] demonstrated that when positive externalities from concentration are strong relative to the negative externality of interaction at a distance, the uniform distribution of activity becomes unstable. They did not examine exactly what occurs when such a threshold of instability is reached. This was taken up by Anas [1, 3] who explained how agglomeration and city formation evolve over time in a simple general equilibrium economy with a closed labor market and positive externalities in localized production. He showed that the uniform distribution of activity is unstable at the early stages of growth when activity concentrates in one or several places. Population growth eventually exhausts scale economies from concentrated activity and centers become uniformly distributed.

This paper is organized as follows. Section 2 describes the model structure by considering the competitive partial equilibrium of the consumer, the firm, and the transport sector and then combines these into a general equilibrium formulation which has three central features: (a)

[5] For example, Beckmann's model is of a city with only households who make social visits to one another. Boruckov and Hochman's model is of a CBD as a collection of firms that interact with one another.

[6] The Fujita and Ogawa model is partial equilibrium: labor and product markets (and so wages and commodity prices) are exogenous, transportation does not take up land and transport costs are exogenous and there is rent leakage (i.e., landlords are absentee).

[7] The model of Harris and Wilson [17] is a good example of the attention to endogenous agglomeration among non-economists. Although their model does not include prices and does not have a transparent economic structure, it does illustrate the basic insight by generating multiple subcenters the number of which is determined by a trade-off between scale economies and transportation costs.

interindustry trade directly links the locations where the various commodities are produced; (b) the shopping trips of consumers connect the locations where commodities are produced and sold; and (c) traffic congestion determines the cost of travel which results from commuting, shopping, and interindustry freight. Scale economies in shopping are then introduced into this basic model.

Section 3 presents numerical solutions for two cases of the general model for a bounded linear city where only one commodity is produced. In the first case, there are no scale economies in shopping and no interindustry trade. An equilibrium land use pattern is determined in which production and residences are dispersed throughout the linear city. Rent, wage, and commodity price gradients peak at the center of the linear space as do land use densities, traffic volumes, and land allocation to roads. This case is examined more extensively in Anas and Kim [4].

In the second case, scale economies in shopping are introduced. The resulting multiple equilibria are examined. If these scale economies are sufficiently powerful relative to the level of congestion in travel, the dispersed production pattern becomes unstable and monocentric or polycentric concentrations of production emerge as alternative equilibria under the same parameter values. Sufficiently large perturbations in the distribution of productive activity will jog the urban system from one polycentric equilibrium to another. The number of centers trades off benefits from agglomeration against benefits from accessibility to centers. For example, when traffic congestion is high a larger number of centers reduces average travel cost and, hence, increases accessibility but at the expense of less agglomeration in production. Conversely, when the level of congestion is sufficiently low, the complete concentration of production (monocentric equilibrium) gives the highest welfare.

Section 4 ends with a road map of further applications of the model.

2. STRUCTURE OF THE MODEL

Space is divided into differentiated locations (zones). The zones are linked via the usual link–node network and all travel between zones takes place on a minimum network path. In our general model, there are a predetermined number of industries which can locate in any subset of zones. Production in each industry takes place by using labor, land, and any subset of the commodities as intermediate inputs. Technology is constant returns to scale. Hence, the number of firms is indeterminate but aggregate industry output is determinate.

Commodities of the same class (i.e., outputs of the same industry) produced in different locations are product variants but commodities of the same class produced in the same location are undifferentiated. This assumption means, in effect, that product differentiation is caused directly by locational differentiation.[8] There are possibly many producers in the same location, hence pricing of each commodity is competitive at each location but varies among locations at equilibrium. Commodities are sold to shoppers and to other firms (as intermediate inputs) at the sites of their production.[9]

We consider a single consumer type but we allow idiosyncratic taste heterogeneity for work–home location choices. Consumers supply labor to the firms at some location, buy land (for homes) at some location, and travel from home *to shop at all locations*, buying their demanded quantities of each differentiated good at each location. This pattern of shopping occurs because the consumer considers goods purchased from each location as essential commodities.[10] One unit of a single commodity is purchased per each shopping trip made. Consumers also determine their leisure demands and choose their work–home location pair.

Land is required for housing and for production, but also for roads which are designed to accomodate shopping and commuting trips and the shipments of intermediate commodities. Travel is congested and thus the money and time costs of travel are endogenous as functions of the allocation of land to roads.

All markets clear so that wages, rents, and commodity prices are determined at each location and land is allocated to roads in such quantities that the congestion tolls collected from all traffic traversing a zone just cover the rental value of the land in roads. All rents are redistributed equally among the households. Our notation is as follows. We will normally use i to denote the residential use of a location, j the use for employment, and k the use for shopping. However, when no conflict is implied among these three uses we will use i to denote a zone. We will use $r = 1 \ldots R$ to denote the commodities.

2.1. The Consumer

The consumer takes as given the distribution of employment and the shopping locations. Consumers are price takers in all markets and take as given all transport costs and travel times. The consumer chooses the pair of work–home locations (j, i), the size of the residential lot, leisure hours, labor hours, and the shopping trip pattern.

[8] The assumption means, for example, that two McDonalds' located at different places are treated as supplying variants of the same ''commodity'' even though the underlying commodities (excluding the location attribute) are identical.

[9] Because production sites and shopping sites are identical we will normally refer to them as shopping sites when discussing consumer behavior and as production sites when discussing firm behavior.

[10] In the context of footnote 8, this means also that each consumer will want (over a period of time) to visit each McDonald's.

The correct way to solve this problem is to recognize that it decomposes into a two-stage problem. Suppose that the work–home location pair is chosen, then consumer choices of land, leisure and shopping trips should be determined in an inner stage maximization subject to the budget constraint which is conditional on the cost of commuting. In the outer stage, the consumer compares the optimized utilities for all work–home pairs and chooses the best.

In the Cobb–Douglas utility function given below by (1), we assume that the taste coefficients α_{rk}, β, and γ are identical across consumers and that $\sum_{rk} \alpha_{rk} + \beta + \gamma = 1$ (homogeneity of degree one). The most striking feature of (1) is that making shopping trips to each location is, as a matter of taste, an essential activity.[11] Z_{ijrk} is the quantity of the rth commodity purchased at production zone k at price p_{rk} per unit. It is also the number of shopping trips made by a worker, employed at j, from the home zone i to zone k to purchase commodity r. q_{ij} is the lot size of the consumer (at home zone i), and L_{ij} is the leisure time of the consumer. ρ_i is the rent for land at i, and w_j is the hourly wage at j. H is the total hours available for work, leisure, and travel per period, and v is the work days (or one-way commutes) per period (year). t_{ik} is the money cost of one-way travel from i to k. T_{ij} is the total travel time per period. g_{ik} is the one-way travel time of a shopping trip from home zone i to k, and $2v$ is the number of commutes per period. D is a rent dividend and u_{ij} are idiosyncratic utility constants.

The inner stage maximization problem is

$$\underset{[q_{ij}, L_{ij}, \mathbf{Z}_{ij} | (i,j)]}{\text{Maximize}} \quad U_{ij} = \textstyle\sum_{rk} \alpha_{rk} \ln Z_{ijrk} + \beta \ln q_{ij} + \gamma \ln L_{ij} + u_{ij} \tag{1}$$

subject to:

$$\textstyle\sum_{rk} Z_{ijrk}(p_{rk} + 2t_{ik}) + \rho_i q_{ij} + 2v t_{ij} = w_j\left(H - T_{ij} - L_{ij}\right) + D. \tag{2}$$

The budget constraint, (2), may be rearranged to express the expenditure of full economic income:

$$\begin{aligned} &\textstyle\sum_{rk} Z_{ijrk}\left(p_{rk} + 2t_{ik} + 2w_j g_{ik}\right) + \rho_i q_{ij} \\ &\qquad + 2v\left(t_{ij} + w_j g_{ij}\right) + w_j L_{ij} = w_j H + D. \end{aligned} \tag{2$'$}$$

[11] In a more general formulation, consumer tastes could be modeled using the C.E.S. utility function. This approach would allow us to specify a preference for variety (see Dixit and Stiglitz [10]) without imposing the assumption that travel to all locations is an essential activity. This becomes important when the urban space grows spatially. Then, with Cobb–Douglas tastes, consumers would never visit the new locations (since these do not enter the utility function and are thus inessential). With C.E.S. tastes, consumers can decide endogenously which places to visit and which not to visit, and since a taste for variety is implied they will in fact choose to visit all locations including the new ones. In our current formulation, the number of locations is fixed and, hence, the use of the C.E.S. over the Cobb–Douglas does not add much generality.

Looked at this way, the consumer has full income $w_j H + D$ and spends this on the RI consumption goods, on land for a home, on commuting, and on leisure. Leisure and travel are "bought" at the opportunity cost of time, which is the wage rate, by working less and thus foregoing potential income. Equation (2′) also shows the full price of trips. For a two-way shopping trip, the full price is $p_{rk} + 2t_{ik} + 2w_j g_{ik}$, and for a one-way commute it is $t_{ij} + w_j g_{ij}$.[12] Total travel time is calculated as

$$T_{ij} = 2\upsilon g_{ij} + \Sigma_{rk} 2 g_{ik} Z_{ijrk}, \tag{3}$$

where $2Z_{ijrk}$ is the number of one-way shopping trips per period.

Consumers are equal owners of all the land in the economy. Hence, aggregate land rents are redistributed as the rental dividend, D. Letting N be the exogenous number of consumers and A_i the amount of land in zone i, the dividend is

$$D = (\Sigma_i A_i \rho_i)/N. \tag{4}$$

The terms u_{ij} in (1) are idiosyncratic taste constants for work–home zone pair (j, i) which, unlike the rest of utility, obtain different values for each consumer. We will return to the role of these below when we discuss the outer stage maximization. In the inner stage, since (j, i) is given, the u_{ij} are given constants.

Because of the Cobb–Douglas utility, maximization will give the usual demand functions with unitary own-price and income elasticities and zero cross-price elasticities with the utility coefficients measuring the constant cost-share of each commodity. These cost-shares are the portion of full income net of commuting allocated to land, leisure, and each commodity. Full income net of commuting for the work–home pair (j, i) is $w_j H - 2\upsilon(t_{ij} + w_j g_{ij}) + D$.

For each commodity r purchased in zone k, the demanded quantity of that commodity and the demanded number of shopping trips are

$$^*Z_{ijrk} = \alpha_{rk} \frac{w_j H - 2\upsilon\big(t_{ij} + w_j g_{ij}\big) + D}{p_{rk} + 2t_{ik} + 2w_j g_{ik}}. \tag{5}$$

The quantity of residential land demanded at i is

$$^*q_{ij} = \beta \frac{w_j H - 2\upsilon\big(t_{ij} + w_j g_{ij}\big) + D}{\rho_i}. \tag{6}$$

[12] "Full income" means the value of all dividends plus the value of the total time endowment. A "full price of a trip" is the delivered price of a unit of the commodity purchased on that trip plus the value of the time spent on the trip.

The leisure demanded is

$$*L_{ij} = \gamma \frac{w_j H - 2v\left(t_{ij} + w_j g_{ij}\right) + D}{w_j}. \tag{7}$$

The optimized or indirect utility can now be found by plugging the demands given by (5)–(7) into (1) and simplifying. We get

$$*U_{ij} = \ln\left[w_j H - 2v\left(t_{ij} + w_j g_{ij}\right) + D\right] - \Sigma_{rk}\,\alpha_{rk} \ln\left(p_{rk} + 2t_{ik} + 2w_j g_{ik}\right)$$
$$- \beta \ln \rho_i - \gamma \ln w_j + u_{ij}. \tag{8}$$

Equation (8) gives the maximum utility possible conditional on the prior choice of the work–home location pair (j, i).

The best work–home pair is found in the outer stage maximization by comparing the indirect utilities of all (j, i) pairs. At this stage, consumers choose differently because the idiosyncratic taste constants for work–home location pairs differ among consumers and are distributed randomly among them. Hence, consumer choices will be determined up to a choice probability. To derive a workable model, we will resort to multinomial logit choice probabilities. These are[13]

$$\Psi_{ij} = \frac{\exp\left(\lambda V_{ij}\right)}{\Sigma_s \Sigma_m \exp\left(\lambda V_{sm}\right)}, \Sigma_{ij}\Psi_{ij} = 1. \tag{9}$$

Ψ_{ij} is the joint probability that the consumer chooses a home at i and a job at j. Note that λ links the dispersion of consumer choices to the variance of the idiosyncratic tastes. At one extreme, as $\lambda \to \infty$, taste heterogeneity vanishes and all consumers choose identically. In this case, the Ψ_{ij} which corresponds to the highest non-idiosyncratic utility goes to unity and all other Ψ_{ij}'s go to zero.[14] Therefore, for all work–home pairs to be chosen at equilibrium, when λ is infinite, rents, wages, commodity prices, and travel times and costs must adjust in such a way that the non-idiosyncratic utilities of all (j, i) pairs are equal at equilibrium. At the other extreme, as $\lambda \to 0$, idiosyncratic taste heterogeneity swamps the systematic parts of utility, given by the V_{ij}'s, and consumers choose randomly so that each $\Psi_{ij} = 1/I^2$, where I^2 is the number of work–home pairs.

[13] We assume, as usual, that the u_{ij}'s are i.i.d. according to the Gumbel distribution with $E[u_{ij}] = 0$, variance σ^2 and dispersion parameter $\lambda = \pi/\sigma\sqrt{6}$. Then, letting the non-idiosyncratic part of indirect utility (8) be defined as $V_{ij} \equiv E[*U_{ij}]$, we derive the logit probabilities as $\Psi_{ij} \equiv$ Prob. $[V_{ij} + u_{ij} > V_{sm} + u_{sm}$, for all (s, m) other than $(i, j)]$. See, for example, Anas [2].

[14] If two or more work–home pairs are tied for maximum utility, they are each chosen with equal probabilities and all other pairs have zero probabilities.

To summarize, we have derived the Marshallian demand functions governing the consumer's behavior together with the probabilities describing consumer dispersal among work–home pairs. It will be notationally convenient for our discussion of general equilibrium to express these in compact form as functions of wages $\mathbf{w}$, rents $\boldsymbol{\rho}$, and prices $\mathbf{p}$, suppressing the transport costs, the rent dividend and all other parameters[15]

$$*Z_{ijrk} = Z_{ijrk}(w_j, p_{rk}), \qquad *q_{ij} = q_{ij}(w_j, \rho_i), \qquad *L_{ij} = L_{ij}(w_j),$$
$$\Psi_{ij} = \Psi_{ij}(\mathbf{p}, \boldsymbol{\rho}, \mathbf{w}).$$

2.2. *The Firm*

Given the location at which it operates, the firm is a price taker in the output and input markets and takes the transport costs of its intermediate inputs as given. Each firm decides how much labor and land and what intermediate input quantities to demand from all other locations. We assume that the firm uses as distinct inputs all the variants of the same commodity produced at different locations (i.e., each product variant is an essential input in production). Because technology is constant returns to scale, optimized profits are normal and there are no economic profits.

We specify the production function of a firm producing commodity r in location j as Cobb–Douglas.[16] Letting X_{rj} be the industry output, M_{rj} the labor input, Q_{rj} the land input, and Y_{rjsn} the quantity of good s from location n used as an intermediate input, the production function is

$$X_{rj} = M_{rj}^{\delta_r} Q_{rj}^{\mu_r} \prod_{sn} Y_{rjsn}^{\phi_{rsn}}, \tag{10}$$

with $\delta_r + \mu_r + \sum_{sn} \phi_{rsn} = 1$. The firm maximizes its profit function with respect to each input quantity

$$\underset{[M_{rj}, Q_{rj}, \mathbf{Y}_{rj}]}{\text{Maximize}} \ \pi_{rj} = p_{rj} X_{rj} - M_{rj} w_j - Q_{rj}\,\rho_j - \sum_{sn} Y_{rjsn}\big(p_{sn} + m_s t_{nj}\big), \tag{11}$$

where m_s is the passenger-equivalent quantity of a unit of commodity-s freight. The input demands (conditional on output level) which solve this maximization are determined by the property that the production function coefficients are the cost-shares of the corresponding input. The input

[15] But, of course, transport costs and times and the rent dividend will be endogenous in the general equilibrium, as we shall see.

[16] Again, as in the case of the consumer, a C.E.S. production function would be more general, but little is gained from using C.E.S., given the purposes of this paper.

demands are

$$M_{rj} = \delta_r p_{rj} w_j^{-1} X_{rj}, \tag{12}$$

$$Q_{rj} = \mu_r p_{rj} \rho_j^{-1} X_{rj}, \tag{13}$$

$$Y_{rjsn} = \phi_{rsn} p_{rj} (p_{sn} + m_s t_{nj})^{-1} X_{rj}. \tag{14}$$

Because of constant returns to scale, costs increase linearly with output (i.e., marginal and average costs are equal and constant) and the firm makes zero profit at any scale of operation. From this property, the price of the output can be expressed as a function of input prices (including its own price, since commodities can be used as inputs in their own production):

$$p_{rj} = \frac{w_j^{\delta_r} \rho_j^{\mu_r} \prod_{sn} (p_{sn} + m_s t_{nj})^{\phi_{rsn}}}{\delta_r^{\delta_r} \mu_r^{\mu_r} \prod_{sn} \phi_{rsn}^{\phi_{rsn}}}. \tag{15}$$

Hereafter, we abbreviate (15) as $p_{rj} = p_{rj}(w_j, \rho_j, \mathbf{p})$. Also, the conditional input demands given by (12)–(14) are denoted as $Q_{rj}(p_{rj}, \rho_j, X_{rj})$, $M_{rj}(p_{rj}, w_j, X_{rj})$ and $Y_{rjsn}(p_{rj}, p_{sn}, X_{rj})$.[17]

2.3. *Transport and the Congestion Externality*

We now turn to our partial equilibrium model of the transport sector. One way to model the behavior of the transport planner is to assume that the planar achieves a first-best optimum. In such a first-best optimum, congestion is priced so that the expected utility level of the consumer is maximized with any profits from road operations distributed to consumers or any losses covered from the aggregate land rent. Such a model of transport planning is strong in that it assumes knowledge of consumer utilities. Furthermore, the issues which we wish to study in this paper center around polycentricity and are not affected much by second-best treatments of road planning.

We model the transport planner, in more realistic terms, as a regulated public authority. In the short run, given the fixed amount of land for roads in a zone, the transport planner is required to levy a congestion toll on each unit of traffic so that transportation is priced at its social marginal cost. Also, the transport planner is required to compensate land owners in a zone by paying the local market price for land taken for roads. While in the short run (with fixed land in roads) the planner may not be able to balance his budget, in long run equilibrium, road operations in each zone

[17] Again, in these abbreviations, transport costs are suppressed for notational simplicity but will be endogenous in general equilibrium.

are required to break even and this is insured by the planner taking an appropriate amount of land in each zone.[18] Even though this behavior is not first-best, we refer to it as "efficient," since it has a strong qualitative similarity to the first-best solution and differs markedly from actual inefficient road planning.

There are three kinds of daily traffic flows originating at a zone i and terminating at the same or another zone j. These, given below, are the daily expected flows of commuters (16), shoppers (16′) and of passenger-equivalent interindustry freight (16″):[19]

$$^{w}F_{ij} = N\Psi_{ij}, \tag{16}$$

$$^{s}F_{ij} = (N/v)\Sigma_{rs}\Psi_{is}Z_{isrj}, \tag{16′}$$

$$^{f}F_{ij} = (1/v)\Sigma_{rs}m_s Y_{rjsi}. \tag{16″}$$

The total daily flow of traffic from i to j is found by summing the above so that $F_{ij} \equiv {}^{w}F_{ij} + {}^{s}F_{ij} + {}^{f}F_{ij}$. When $i = j$ the flows are intrazonal.[20]

We now turn to a specific geography. The I zones of the urban area are rectangles of a narrow width and of unit length, arranged linearly as a discretized version of Solow and Vickrey's [31] "long narrow city." We number the zones consecutively from one end to the other as $1 \ldots I$. We let g_i and t_i denote the travel time and monetary cost of traversing the length of i. Intrazonal trips are assumed to travel half a zone length, and trips terminating at a zone traverse half of it (on average) by the convention that all activities are uniformly spread within a zone.

Then, for intrazonal trips $g_{ii} = (1/2)g_i$ and $t_{ii} = (1/2)t_i$, and for interzonal trips from i to j: $g_{ij} = (1/2)(g_i + g_j) + \Sigma_{s=i+1}^{j-1} g_s$, and $t_{ij} = (1/2)(t_i + t_j) + \Sigma_{s=i+1}^{j-1} t_s$.

[18] As is well known, if the production function for road capacity is constant returns, then the congestion toll just covers the land rent for each road segment. In the case of increasing returns, there is a deficit and in the case of decreasing returns, a surplus.

[19] Note that the flows (16) and (16′) are constructed by multiplying the total number of travelers N, with the probability that the traveler will choose home–work pair (i, j); and in (16″) also multiplying by the number of shopping trips and appropriately summing over all industries and work locations. Division by v converts annual shopping trips to daily. In (16″) multiplication by m_s converts commodity flows to passenger equivalent units.

[20] From an empirical standpoint adding commuting, shopping, and freight flows is inappropriate since these flows will differ sharply by time of day. However, in this theoretical model we suppress trip-timing and pretend that all travel occurs simultaneously. It is equally simple to pretend that each type of travel is segregated in time from the others. The case where they overlap is the most difficult to treat.

The total flow traversing the edge zones $i = 1$ or I is

$$F_i = F_{ii} + \sum_{j \neq i} (F_{ij} + F_{ji}). \tag{17}$$

For an internal zone $2 \leq i \leq I - 1$, it is

$$F_i = F_{ii} + \sum_{j \neq i} (F_{ij} + F_{ji}) + 2\left\{ \sum_{j=1}^{i-1} \sum_{k=1}^{I-i} (F_{i-j,\,i+k} + F_{i+k,\,i-j}) \right\}, \tag{17'}$$

where the term in $\{\cdot\}$ is the through-traffic crossing the zone and is multiplied by 2 because through traffic crosses the entire zone.

Now we turn to congestion. Suppose that traffic capacity K_i in zone i is given by the function $K_i = \zeta(S_i)^{\omega}$, where $0 < \omega < 1$ and S_i is the amount of land placed in roads in zone i. This says that land is the only input for roads and that there are diminishing returns in using land to create road capacity.[21] The time for a passenger to cross the zone is given by the congestion function

$$g_i = a_i \left[1 + b(F_i / K_i)^c \right], \tag{18}$$

where $a, b > 0$ and $c \geq 1$. The total cost incurred by the traffic crossing i is then $G_i \equiv g_i F_i$ and the marginal cost of a unit amount of traffic is

$$\frac{\partial G_i}{\partial F_i} = a_i \left[1 + b(1 + c)(F_i / K_i)^c \right]. \tag{19}$$

The congestion toll, in units of time, is $\tau_i \equiv (\partial G_i / \partial F_i) - g_i = a_i bc (F_i / K_i)^c$. The monetary cost of travel t_i is the congestion toll weighted by the traveler's value of time. We have already assumed that travelers value travel time at their wage rates. Travelers' wages differ as their job locations differ. We assume that travelers cannot be charged tolls according to their specific job locations. We assume that travelers crossing a zone i are charged according to a weighted value of time of all the commuters residing at that zone and working in all zones. With these assumptions, the cost of transport or the congestion toll for traffic crossing i is

$$t_i = a_i bc (F_i / K_i)^c \left(\sum_j F_{ij} w_j / F_i \right). \tag{20}$$

[21] When $\omega = 1$ there is constant returns to scale in using land to create traffic capacity. Since our model ignores non-land inputs in roads, the role of these inputs is proxied by the land inputs. Hence, it would be more appropriate to assume $\omega < 1$ rather than $\omega = 1$ to reflect the fact that when land and non-land inputs are considered, roads are subject to decreasing returns to scale.

As already explained, land is allocated to roads in such a way in each zone i that the total toll revenue collected from all the traffic crossing the zone pays for the rent of the land in roads in that zone. Hence

$$S_i \rho_i = \upsilon F_i t_i. \tag{21}$$

Using (20) and the land-capacity relation $K_i = \zeta(S_i)^\omega$ we solve (21) for the long run equilibrium road capacity in zone i:[22]

$$K_i^* = \left\{ \frac{\upsilon a_i bc F_i^c \zeta^{1/\omega} \left(\Sigma_j F_{ij} w_j \right)}{\rho_i} \right\}^{\omega/(1+c\omega)}. \tag{22}$$

2.4. General Equilibrium

We now combine the three partial equilibrium models into a general equilibrium model. The number of consumers, N, the total time endowment of each consumer, H, and the amount of land, A_i, in each zone are given. In the linear city $A_i = A$ for each i. General equilibrium finds commodity prices (by zone and commodity) $\mathbf{p}$, land rents, $\boldsymbol{\rho}$, and wages, $\mathbf{w}$ (by zone), industry outputs (by zone and commodity) $\mathbf{X}$, and the equilibrium congestion tolls, $\mathbf{t}$, and travel times, $\mathbf{g}$ (by zone).

Equilibrium in the land market in i requires

$$N \Sigma_j \Psi_{ij}(\mathbf{p}, \boldsymbol{\rho}, \mathbf{w}) q_{ij}(w_j, \rho_i) + \Sigma_r Q_{ri}(p_{ri}, \rho_i, X_{ri}) + S_i = A_i. \tag{23}$$

The left side is the sum of the lot size demands of all households residing in zone i and commuting to all zones plus the land demands of all the firms in i plus the land allocated to traffic. The right side is the available land in i. In the labor market in i

$$N \Sigma_s \Psi_{si}(\mathbf{p}, \boldsymbol{\rho}, \mathbf{w}) \left[H - T_{si} - L_{si}(w_i) \right] = \Sigma_r M_{ri}(p_{ri}, w_i, X_{ri}). \tag{24}$$

Here, the left side is the supply of labor by consumers working in i and the right side is the demand for labor by all the firms producing in i. The total travel time T_{si} (see (3)) is given by the transport sector equilibrium. In the market for commodity (r, i)

$$N \Sigma_{ns} \Psi_{ns}(\mathbf{p}, \boldsymbol{\rho}, \mathbf{w}) Z_{nsri}(w_s, p_{ri}) + \Sigma_{ns} Y_{nsri}(p_{ns}, p_{ri}, X_{ns}) = X_{ri}. \tag{25}$$

The left side is the quantity of commodity r shopped in zone i by consumers who work and live in all the zone pairs plus the demand for the same commodity for use as an intermediate input by the firms producing

[22] Note that condition (21) need not be imposed but will hold automatically when road capacity is constant returns to scale ($\omega = 1$).

in all the zones (i.e., interindustry trade). The right side is the industry output of the rth commodity in zone i. The last set of equations is the already-derived (15), the relationship among output and input prices implied by the zero-profit equilibrium of the firm producing commodity r in i

$$p_{ri} - p_{ri}(w_i, \rho_i, \mathbf{p}) = 0. \tag{26}$$

The total number of Eqs. (23)–(26) is $2I + 2RI$. The number of unknowns to be found by solving these simultaneously are $\mathbf{p}$, $\mathbf{X}$, $\mathbf{w}$, and $\boldsymbol{\rho}$, also $2I + 2RI$ in number.

This general equilibrium system is homogeneous of degree zero in all prices, $\mathbf{p}$, rents, $\boldsymbol{\rho}$, and wages, $\mathbf{w}$.[23] Hence, equilibrium prices can be normalized and demands can be expressed in terms of relative prices. We adopt a convention that the rent for land at the last zone is fixed and, hence, we discard the land market condition for the last zone.[24]

2.5. *Shopping Externalities*

We now build scale economies in shopping into the model. We assumed that the consumer's cost-share coefficients, the α_{rk}'s, were given preference constants which varied by the type and place of purchase.

Suppose, instead, that cost shares are defined as α_{rk}'s, where i is the home location of a worker, r is the commodity, and k is the shopping place. We assume that $\sum_k \alpha_{rk} = \alpha_r$ for each home zone i and that $\alpha_r + \beta + \gamma = 1$. Hence, $\alpha_r = 1 - \beta - \gamma$ for each r. Recall that X_{rk} is the scale of production, also measuring the available volume of product r at k. Suppose that the shopping preference coefficients for commodity r purchased in zone k are given by

$$\alpha_{rk} = \alpha_r \frac{X_{rk}^{\eta}}{\sum_j X_{rj}^{\eta}}. \tag{27}$$

Equation (27) guarantees that $\sum_k \alpha_{rk} = 1 - \beta - \gamma$ for all home locations i which means that the consumer spends a fixed proportion of income to purchase commodity r throughout all shopping centers, regardless of

[23] The proof of homogeneity is included in the working paper by Anas and Kim [4], an earlier version of this paper which can be obtained from Alex Anas, but is excluded from this paper to save space.

[24] In all partial equilibrium urban models it is customary to assume that the city is surrounded by agricultural land and that, at the border, urban and agricultural rents will be equal. Since our model is completely closed, there is no agriculture (or it is included in the commodity set) and it is more appropriate to think of the city as an island. Therefore, the fixed rent at the edge zone does not reflect an assumption of an exogenous land rent, but it reflects the fact that, in general equilibrium, one price is arbitrary, because of Walras's Law.

home location. By $\eta > 0$ we have the property that each consumer likes larger shopping centers and, hence—because of Cobb–Douglas tastes—spends more income on a given good r at a shopping zone that sells more of that good, keeping the home zone fixed.

The idea that shopping center sizes (in our case size is measured by the outputs X_{rk}) determine consumer propensity to spend appears to have had empirical support. The shopping models developed by Huff [21], Lakshmanan and Hansen [24], and Harris and Wilson [17] embodied similar concepts. Although the income share spent on a shopping center is determined by the center's relative size, the full set of relevant economic variables also affects *quantity* purchased. To see this, we plug (27) into the Marshallian demand function for the commodity rk given by (5).

$$*Z_{ijrk} = \alpha_r \left\{ \frac{X_{rk}^{\eta}}{\sum_j X_{rj}^{\eta}} \right\} \frac{\left[w_j H - 2v\left(t_{ij} + w_j g_{ij}\right) + D\right]}{\left(p_{rk} + 2t_{ik} + 2w_j g_{ik}\right)}. \tag{28}$$

The above equation makes it clear that the relative size given by $\{\cdot\}$ acts as a shift factor in the Marshallian demand function, but the quantity purchased is also influenced by disposable income and by the full price of a shopping trip as is seen from the second part of (28).

In terms of the general equilibrium solution, the modified demands given by (28) mean that the industry outputs **X** obtained from the solution must be consistent with the α_{rk}'s used to obtain them. Note also that a proportional change in shopping center sizes preserves homogeneity of degree zero of excess demands.[25]

3. NUMBERS OF SUBCENTERS, STABILITY AND WELFARE

There are three major economic relationships included in the model which could be suppressed or treated in tailoring numerical applications. These are: (a) traffic congestion; (b) interindustry trade (exchange of intermediate inputs in production); and (c) economies of scale in shopping.

[25] In Anas and Kim [4], we described the computational algorithm in detail. It consists of an iterative procedure for adjusting wages ($\mathbf{w}$), rents ($\boldsymbol{\rho}$), product prices ($\mathbf{p}$) and the allocation of land to roads in such a way that the labor, land and commodity markets clear simultaneously in each location. While this is done, the algorithm also ensures that consumer incomes be consistent with the redistributed rent dividend and that travel times be consistent with consumer demands for travel and the road planner's long run equilibrium conditions. An extended algorithm deals with the case of economies of scale in shopping ($\eta > 0$) by insuring that consumer demands for shopping (given by (28)) are consistent with the sizes of the shopping centers at equilibrium. This extended algorithm is designed to find multiple equilibria and, in the manner described in Section 3 of Anas and Kim [4], to test the stability of each such equilibrium.

Congestion exists by setting $b > 0$. Interindustry trade requires that at least some ϕ's be positive, and there are economies of scale in shopping when $\eta > 0$.

In the applications considered here we suppress interindustry linkages by assuming that there is only one commodity $(R = 1)$ and that production takes place by using land and labor as the only two inputs. Hence, there is no interindustry trade and no freight flows. More generally (with more than one commodity) the absence of trade is insured by all $\phi_{rsn} = 0$.[26] Other parameters are set as follows. We divide the linear city into $I = 11$ identical zones. Zone 6 is the central zone. Arranged symmetrically around this zone, zones 5 and 7, 4 and 8, 3 and 9, 2 and 10, and the edge zones 1 and 11 are identical *a priori*. Zone width is set at 0.1 miles and zone length is one mile.[27] The fixed rent in zone 1 (or 11) is set at $10 per square yard. There are $N = 1000$ households. Each has a total time endowment of $H = 260$ hours and the number of working days per period (per month) is $v = 22$. The income share of land $\beta = 0.35$ and the income share of leisure $\gamma = 0.15$. The share of all other consumption is $\alpha = 0.50$. Note that the subscript r on α is suppressed since $R = 1$. In the case without scale economies in shopping $(\eta = 0)$, analyzed in Anas and Kim [4], we assumed that the consumer is, *ceteris paribus,* inclined to spend the same portion of income in every zone (i.e., likes all shopping locations equally). Then, $\alpha_k = \alpha/I = \alpha/11\ (= 0.04545\dots)$. The parameters in the congestion function are set as $a = 1$, $b = 0.15$, and $c = 4$ and road technology is defined by $\zeta = 7.0$ and $\omega = 0.8$. The taste heterogeneity coefficient for the home–work location choice is $\lambda = 1$. The labor and land elasticities of output given by δ and μ sum to one.

With the above parameter values, we analyzed two cases. In the first of these (Anas and Kim [4]), we assumed that shopping preferences are not affected by the size of a shopping site $(\eta = 0)$. Then, congestion is the only externality present. This generates a unique equilibrium solution in which consumers and jobs are dispersed among all zones of the linear city and in which rent, wage, commodity price gradients, and traffic all peak at the central zone.[28]

[26] The case of interindustry linkages requires extensive treatment and is left to another paper.

[27] To find equilibrium in this 11-location city, the algorithm solves simultaneously 33 equations (labor, land, and commodity excess demands for each location), of which one is redundant by Walras's Law, together with the additional conditions for transport equilibrium, income composition, and (when $\eta > 0$), consistency between shopping demands and shopping center sizes.

[28] This "dispersed city" case is analyzed in (Anas and Kim [4]) by performing comparative statics on the dispersed equilibrium. The parameters N, λ, and δ are varied and the sensitivity of endogenous variables (especially the land use densities and the three gradients) on these parameters is explored.

In the second set of cases, to be discussed below, we assumed that $\eta > 0$. These cases generate polycentric urban structures: there are multiple equilibria under the same parameter values. The nature of these equilibria is such that employment may be concentrated in a subset of the zones while residences and streets are present in all zones. In the dispersed equilibrium (Anas and Kim [4]) spatial concentration occurs around the central zone even though scale economies in shopping are suppressed ($\eta = 0$). This occurs because of the accessibility advantages of the central zone. When $\eta > 0$, there is an additional factor due to the scale economy in shopping.

In this case, as we saw, consumers' preferences for shopping depend not only on their disposable incomes and the full prices of shopping trips but also on the relative sizes of the shopping destinations. *Ceteris paribus*, consumers spend more disposable income at larger shopping destinations. More precisely, suppressing r in (27) we can find that $\partial\alpha_i/\partial X_i = \eta\alpha_i(1 - \alpha_i)/X_i > 0$, which says that the share of income spent on shopping in zone i increases with the production-size (or shopping center size) of zone i. Hence, when $\eta > 0$, shopping center size confers a positive effect on shopping center revenue.

There are multiple equilibria in the sense that, at some equilibria, production may concentrate in some zones only while the other zones are reserved for residences (streets are also present in all zones). We will call an equilibrium in which some zones contain no production a *polycentric equilibrium* (the monocentric case is included as a special case). Furthermore, such equilibria may be spatially symmetric or asymmetric. Below, we will focus only on symmetric equilibria.

When multiple equilibria are present, stability becomes important in deciding the likelihood that a particular equilibrium will be sustained (if it occurs). To perform stability analysis it is necessary to construct a model of how the system adjusts when it is out of equilibrium. Our computational algorithm, which mimics market price adjustments in response to excess demands is, in fact, such a dynamic adjustment procedure. Therefore, a particular equilibrium will be stable (unstable) if application of our algorithm from a particular non-equilibrium state converges the economy on that (diverges it away from that) polycentric or monocentric equilibrium.

The presence and stability of a polycentric equilibrium depends crucially on the value of η which controls the strength of the scale economy. To understand how polycentricity works, we consider the following simple but insightful example. Suppose that there are three zones with a single unit of output in each ($X_1 = X_2 = X_3 = 1$). Suppose, for the sake of this example, that all other variables, prices included, are kept constant. We will consider what happens if the output of zone 2 is moved to zone 1 so that $X_1 = 2$, $X_2 = 0$, and $X_3 = 1$. Suppose that $\eta = 1/2$ (more generally $\eta < 1$), then moving production from zone 2 to zone 1 increases the value of α_1 from a third to 0.583. Clearly, there is no incentive for the production in zone 2 to move to zone 1 since this doubles the aggregate output of zone 1 while less than doubling the aggregate revenue received at the zone. Hence, with $\eta < 1$ the external scale economies in consumption are not strong enough to induce agglomeration of producers.

When $\eta = 1$, then moving output from zone 2 to zone 1 exactly doubles revenue collected in zone 1. When $\eta > 1$, then revenue received in zone 1 more than doubles. Hence, we should expect the dispersed employment pattern to become unstable and a polycentric pattern to become stable if the external scale economies in consumption are sufficiently strong.

The value of η must be sufficiently bigger than one to overcome the other influences which encourage the dispersed pattern. High transport costs encourage dispersion: the consumer wants to conserve travel, hence producers become dispersed among the consumers to reduce travel times and costs incurred in shopping. Therefore, when η is sufficiently high relative to the cost of congested travel, then polycentric patterns of land use emerge as stable equilibria and the dispersed patterns become unstable.

Consider now the way our algorithm finds the polycentric equilibria and ascertains computationally whether they are stable or not. More precisely, we wish to ascertain ‘‘*how stable ?*’’ a given polycentric equilibrium is, since as long as multiple equilibria are present a large enough perturbation would destabilize any locally stable equilibrium and ‘‘jog’’ the economy to another equilibrium. Then, ‘‘*how stable ?*’’ means: ‘‘how big a perturbation must the economy be shocked with to switch from one equilibrium to another?’’

We will confine our attention to the symmetric polycentric equilibria in our 11 zone linear city. These are: the 1-peak (monocentric with all production in zone 6); the 3-peak (tricentric with production in zones 3, 6, and 9); and the 5-peak (pentacentric with production in zones 2, 4, 6, 8, and 10). Any polycentric equilibrium is easy to find via the computational algorithm reported in Section 3 of Anas and Kim [4]. Suppose that we wanted to find the monocentric case. We would force all $X_i = 0$ for i other than 6. We would iterate subject to these constraints until the monocentric case was confirmed to be an equilibrium upon convergence of the algorithm. Similarly, production would be forced to zero in zones 1, 2, 4, 5, 7, 8, 10, and 11 if we wanted to find the tricentric equilibrium, with positive production in zones 3, 6, and 9 endogenously determined. In the symmetric equilibrium, zones 3 and 9 will have identical distributions of activity, different from zone 6. Similarly, at equilibrium, zones 1 and 11, 2 and 10, 4 and 8, and 5 and 7 will be pairwise identical.

To confirm the stability of a polycentric equilibrium, we start with that polycentric equilibrium solution and we perturb it systematically. For example, we perturb the monocentric equilibrium by removing a bit of the production located in the center and placing it equally among zones 3 and 9. We iterate with this non-equilibrium starting point. If transport costs are sufficiently high relative to η, only a small perturbation would be sufficient for the monocentric case to decay to the tricentric case and the algorithm would converge on this pattern. Whether the monocentric equilibrium decays to the tricentric case depends not only on transport costs but also on the size of the perturbation. Relatedly, the size of the perturbation needed to achieve the decay of the monocentric equilibrium is smaller, the higher the level of transport congestion.

To see this better, imagine a two-zone example. Suppose that production exists in zone 1 but not in zone 2 and that this is an equilibrium. Since we allow free entry of firms, some investors can perturb the equilibrium by trying to produce in zone 2. Suppose that an investor builds a factory/shopping center in zone 2 which is one hundred times smaller than that in zone 1. Can such an investment succeed? This depends on the consumer's shopping behavior which is driven, in part, by η. If this value is small enough compared to unity, scale economies in shopping are slight and the small amount of production in zone 2 will succeed and may grow. Then, an equilibrium can be found with positive production in both zones. However, if the value of η is sufficiently bigger than one, then production in zone 2 will vanish because consumers strongly prefer bigger places in which to shop and the disturbance created by the investment in zone 2 is not big enough to attract sufficient consumers. Otherwise, if the investment in zone 2 is big enough, production may disappear from zone 1 and appear in zone 2.

To summarize, with a relatively small value of η, the consumer's shopping behavior is more sensitive to the delivered price (which includes travel time and cost) than to the relative size of the shopping center. Therefore, a firm can successfully survive in a zone even with relatively low production, if the firm in the zone can produce and deliver the commodity cheaply. That is why a bigger disturbance (larger shopping center critical size) is required to generate a polycentric pattern when the value of η is relatively small.

Table 1 illustrates the results for the monocentric, the completely mixed (or dispersed), and two other polycentric configurations all of which are equilibria under $N = 1{,}000$, $\delta = 0.9$, and $\eta = 1.7$, and the other parameter values mentioned earlier. Sufficiently big perturbations would move the economy from any one of these equilibria to one of the others. The land use patterns of these four cases are depicted in the bar charts of Figs. 1 and 2.

The completely mixed case should be discussed first. Clearly, it resembles the dispersed case mentioned earlier and discussed in Anas and Kim

TABLE 1
Completely Mixed (C-mix) and Polycentric Equilibria

Zone		6	5 or 7	4 or 8	3 or 9	2 or 10	1 or 11
C-mix	ρ_i	11.045	10.995	10.850	10.621	10.325	10.00
	N_i (%)	9.046	9.051	9.065	9.088	9.119	9.155
	X_i (%)	9.228	9.215	9.173	9.104	9.007	8.887
5-peak	ρ_i	13.450	10.971	13.181	10.597	12.461	10.00
	N_i (%)	8.705	9.338	8.727	9.369	8.791	9.423
	X_i (%)	20.201	0	20.101	0	19.799	0
3-peak	ρ_i	15.143	10.957	10.895	14.410	10.195	10.00
	N_i (%)	8.342	9.325	9.322	8.395	9.398	9.390
	X_i (%)	33.534	0	0	33.233	0	0
1-peak	ρ_i	24.232	10.929	10.652	10.404	10.184	10.00
	N_i (%)	7.083	9.310	9.300	9.291	9.282	9.274
	X_i (%)	100	0	0	0	0	0

Note. $\eta = 1.7$; $\delta = 0.9$; $N = 1{,}000$.

[4]. But it differs from this in that the eleven α's of the consumers are not all identical but now increase with the production level of the zone. More disposable income is spent in the central zone (higher α), because it has a larger shopping center. That accounts for a steeper rent gradient than in the dispersed case. The monocentric case has the highest rent in the center and the steepest rent gradient and that obviously occurs because the demand for land in zone 6 greatly increases as all employment wants to locate there. In the 5-peak and 3-peak cases, the rent at the center (zone 6) is higher than the rent in the secondary centers. The same applies to wages, output, and the output price.

From Table 2 we can discern the effect of increasing η on the completely mixed case relative to the dispersed employment case ($\eta = 0$). Note that as η increases, consumers want to spend more in the central and larger agglomeration. The increased demand for their output induces the centrally located firms to raise their demands for both land and labor and, hence, rents and wages rise. Rents and wages rise less in the secondary centers because as η rises demand for bigger centers increases. Output rises in the center and falls in the edges and residents are decentralized more to make more room for central production.

Also of interest is the welfare ranking of the multiple equilibria which occured under the same parameter values. Which is the maximum welfare equilibrium and how does this change as the population grows exogenously? As population grows, agglomeration effects become more powerful but travel congestion also increases. As we saw, the former effect favors

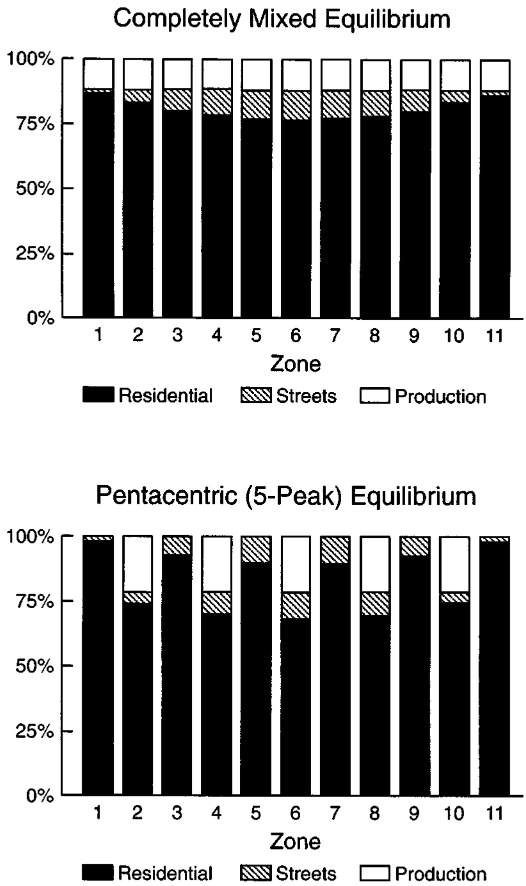

FIG. 1. Land uses in the completely mixed and 5-peak equilibria.

fewer centers whereas the latter effect induces dispersal of central production to more centers. When η is sufficiently large relative to the level congestion, a single center is socially preferred (the consumer's expected utility is highest under a single center). As η gets smaller, equilibria with more centers give higher expected utility and eventually the completely mixed case is the socially preferred equilibrium.

The obvious question "how to induce the economy to produce the optimal numbers and sizes of centers?" is beyond the scope of this paper but easily grasped by the intuition developed via the current model. Clearly, the answer lies in providing tax or other incentives to developers to build larger centers (when equilibria with large centers are socially

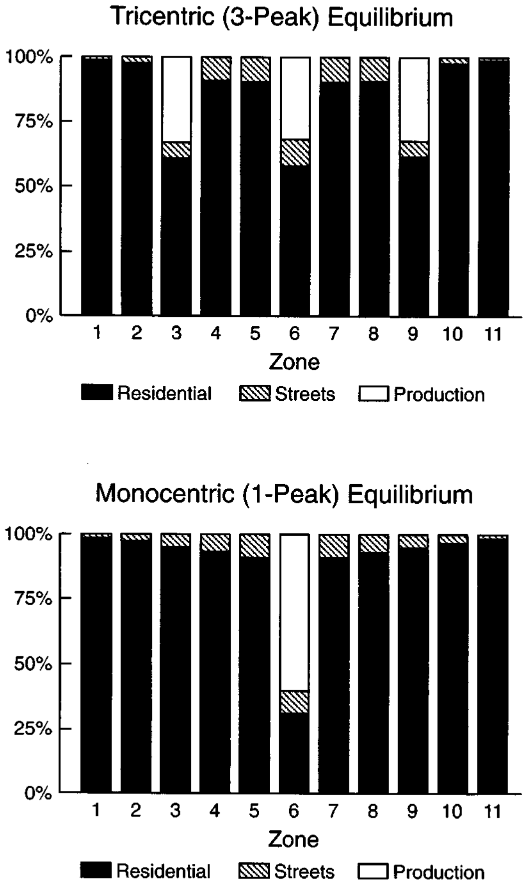

FIG. 2. Land use in the 3-peak and 1-peak (monocentric) equilibria.

preferred). In the absence of such incentives to developers, producers are not motivated to fully internalize the positive externality they confer on shoppers by agglomerating with one another. We might conclude therefore, that the market will produce too few centers and/or smaller than optimal centers when the value of η is high. The economy can get "stuck" in a socially suboptimal but stable equilibrium.

4. A RESEARCH AGENDA

Although the simulations discussed above clearly demonstrate the model's potential, more awaits to be done with this computable model and the purpose of this section is to briefly chart a roadmap for such applications.

TABLE 2

Effects of Varying the Scale Economy in Consumption on the Completely Mixed Case

	η	Zone 6	Zone 3 or 9	Zone 1 or 11
Rent	0.0	10.978	10.582	10.000
	1.4	11.015	10.603	10.000
	1.7	11.045	10.621	10.000
Wage	0.0	19.088	18.916	18.640
	1.4	19.307	18.969	18.409
	1.7	19.481	19.013	18.822
Residents (%)	0.0	9.053	9.089	9.144
	1.4	9.049	9.088	9.150
	1.7	9.046	9.088	9.155
Production (%)	0.0	9.128	9.094	9.037
	1.4	9.184	9.099	8.954
	1.7	9.228	9.104	8.887

Note. $\delta = 0.9$; $N = 1{,}000$.

An interesting theme in urban economics is whether transport improvements are capitalized into rents. While there is plenty of evidence supporting that notion, it is virtually all derived from partial equilibrium models with fixed wages or fixed employment or both in monocentric cities. The present model, therefore, can be applied to examine the relative capitalization of transport improvements into wages, rents, and commodity prices.

Second, interindustry linkages were suppressed in this paper. In a separate paper, interindustry linkages can be explored in depth by introducing more than one commodity. One important question is how having more than one commodity affects rent and wage gradients and which commodities follow a more centralized pattern. Some partial equilibrium analyses of this problem with exogenously specified centers exists (see Hartwick and Hartwick [18]).

Third, we assumed that locations are differentiated but products in the same industry produced at the same location are undifferentiated. It is easy to extend the model to introduce scale economies (or fixed costs) in firm setup and thus make the number of firms explicit and endogenous. Such a framework would also allow the assumption that each firm produces a product variant. Monopolistic competition among firms would determine above-marginal-cost pricing and the aggregate profits of firms would be captured by the consumer−landowners.

Fourth, the shopping technology of consumers can be extended to introduce economies of scale in shopping which stem from multipurpose trips. Suppose that the consumer buys from more than one industry during each shopping trip. Then, firms producing different goods have an incentive to locate together. When firms locate close to each other they confer travel savings to consumers which allows them to price their products higher. If producers cannot locate together, then retailing should emerge endogenously as the market's response to bundling related goods.

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