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Why is central Paris rich and downtown Detroit poor? An amenity-based theory

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Abstract

This paper presents an amenity-based theory of location by income. The theory shows that the relative location of different income groups depends on the spatial pattern of amenities in a city. When the center has a strong amenity advantage over the suburbs, the rich are likely to live at central locations. When the center's amenity advantage is weak or negative, the rich are likely to live in the suburbs. The virtue of the theory is that it ties location by income to a city's idiosyncratic characteristics. It thus predicts a multiplicity of location patterns across cities, consistent with real-world observation. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Although some American central cities have rich enclaves, high-income residents in U.S. urban areas tend to live in the suburbs. This pattern is often reversed, however, outside the U.S. A prime example is the Paris metropolitan area, where the central city has higher average income than the surrounding suburbs. The contrast between the Parisian case and the case of Detroit, a major U.S. metropolitan area, is shown in Table 1. While the Table shows that Detroit's pattern of lower central-city income is repeated (although less dramatically) in other U.S. metropolitan areas, the average French provincial city does not resemble Paris, following the U.S. pattern instead. Notable exceptions, however, are Lyon (the second-largest city), Caen, and Nancy, where incomes are higher in the center.[1] The Parisian pattern is also repeated in other European cities outside France, as noted by Hohenberg and Lees (1986) in their monograph on European urban history. They state that 'incomes rose with distance to the city center in America, whereas they typically fell in Europe' (p. 299). Ingram and Carroll (1981) show that the Parisian pattern also exists in a number of Latin American cities. Location by income thus varies dramatically across countries, and these differences beg for a theoretical explanation.

One of the contributions of the monocentric-city model developed by Alonso (1964), Mills (1967) and Muth (1969) is to give insight into the effect of income on

Table 1
Central-city vs. suburban incomes in France and the US

	Household income[a]	
Case	Central-city	Suburbs
Ile de France (Paris metro area)	124 000 Fr.[b]	106 000 Fr.
Province (other metro areas)	76 000 Fr.	84 000 Fr.
France (all metro areas)	84 000 Fr.	82 000 Fr.
Detroit (metro area)	\$20 207	\$40 084
U.S. (all metro areas)	\$26 727	\$26 314

[a] Household incomes are the 1990 average value in France and the 1989 median value in the U.S. The French data are from Nicot (1996), and the U.S. data are from the 1990 Census.
[b] The current franc-dollar exchange rate is approximately 6 francs per dollar.

[1] See Nicot (1996). Tabard (1993) constructs an index measuring the extent to which highly skilled workers are represented in the work forces of French cities with populations over 50 000. All of the districts of central Paris (each treated as a separate city) are in the upper half of the distribution of this index when its values are ranked for cities in the Paris metropolitan area. This provides further indirect evidence of the concentration of high-income households in central Paris.

household location. Two opposing forces are identified by the model. First, the high housing consumption of the rich means that they are more strongly attracted than the poor by low housing prices in the suburbs. However, rich households also have a high opportunity cost of time and thus a high commuting cost per mile. Therefore, the rich value accessibility to the central business district (CBD) more than the poor. The theory shows that the net effect of these forces hinges on the behavior of the ratio of commuting cost per mile, t, and housing consumption, q. If t/q rises with income, then the accessibility effect dominates, and the rich tend to live at central locations. If t/q falls with income, then the effect of higher housing consumption dominates, and the rich tend to live in the suburbs.

Given this locational indeterminacy, the model is consistent with the variety of location patterns observed in real-world cities. For example, t/q might fall with income in the U.S. while rising with income in France, which would explain the location patterns in the two countries. However, it is unsatisfactory from a scientific point of view to explain real-world complexity by appeal to an ambiguous theory. Moreover, the required difference in the behavior of the t/q ratio across countries seems implausible.[2] Therefore, it is important to consider other explanations for the observed location patterns.[3]

The explanation proposed in the present paper links the location of different income groups to the spatial pattern of amenities in the city. To draw this link, the analysis adopts a key assumption, namely that the marginal valuation of amenities *rises sharply with income*. In addition, it is assumed that the conventional forces discussed above favor suburban location of the rich. Two cases are then considered: amenities fall rapidly with distance to the CBD; amenities fall slowly or even increase with distance to the CBD. In the first case, the amenity advantage of the center along with the high amenity demand of the rich generates a force that pulls the rich toward the center more strongly than the poor. If this force is powerful enough, it can dominate the conventional forces

[2] Wheaton (1977) found that t/q appears to be roughly constant across different income groups in the U.S. This suggests that observed location patterns cannot be explained by appealing to a difference in this ratio across income groups.

[3] LeRoy and Sonstelie (1983) introduce an important modification of the standard model by adding transportation mode choice. They assume that when rich and poor use the same transportation mode, t/q rises with income, so that the rich live in the center. However, if the rich switch to a faster commuting mode (i.e., auto) while the poor continue to use the slow mode (i.e., public transit), t falls for the rich but not for the poor. As a result, the mode switch can make t/q smaller for the rich, leading to a location reversal. While Gin and Sonstelie (1992) present historical evidence from Philadephia that supports this theory (the mode switch in this case was from walking to the streetcar), it is not clear that the theory is capable of explaining the worldwide variation in location patterns. Alternatively, Kern (1981) points to changes in taste as an explanation for reversals in the pattern of location by income.

that pull the rich outward, leading them to locate in the center. However, if the center's amenity advantage is weak or negative, then the amenity force will be insufficient to overcome the conventional forces, and the suburban location of the rich will be maintained.

The urban amenities that underlie this theory can be classified into three categories. *Natural amenities* are generated by an area's topographical features, including rivers, hills, coastline, etc. *Historical amenities* are generated by monuments, buildings, parks, and other urban infrastructure from past eras that are aesthetically pleasing to current residents of the city. While natural and historical amenities are largely exogenous, *modern amenities* are endogenous, with their levels depending on the current economic conditions in a neighborhood, especially the local income level. Such amenities might include restaurants, theaters, and modern public facilities such as swimming pools and tennis courts. Modern amenities may also be linked to historical amenities. This connection arises through the *renovation* of a central city's historical districts, which enhances historical amenities and may be responsive to the current level of income.

The theory sketched above focuses on natural and historical amenities. Since these amenities are *exogenous*, they can be viewed as a causal factor in determining the pattern of location by income. The first case discussed above, where exogenous amenities decline rapidly with distance to the center, might correspond to Paris. Its historical monuments, parks, boulevards, fine architecture, and river scenery give central Paris a large amenity advantage over the suburbs. If the amenity demand of the rich is strong enough, such an advantage might be sufficient to draw rich households to central locations, reversing the U.S. pattern. By contrast, since an American urban area like Detroit lacks the rich history of Paris, the central-city's infrastructure does not offer appreciable aesthetic benefits. This means that no amenity force is working to reverse the conventional forces that draw the rich to the suburbs. As a result, central Detroit is poor. The analysis developing this insight is presented in Section 2 of the paper.

While this theory claims that location by income depends on the city's amenity pattern, it ignores reverse causation. In other words, it overlooks the possibility that urban amenities are a *consequence* rather than a *cause* of the location patterns of different income groups. Amenities with this feature fall into the modern category discussed above, and in the case of Paris, they would include the concentration of excellent restaurants in the central city. While these restaurants constitute a major amenity, their presence is due in part to the high incomes of the local residents, and is not a causal factor explaining those incomes.

Section 3 shows that the major lessons of the exogenous-amenities model of Section 2 are mostly unaffected when endogenous, modern amenities are added to the analysis. This modification is carried out by assuming that neighborhood income also counts as an amenity, capturing the variety of ways in which income influences the quality of life in an area. It is shown that when the rich value the income amenity more highly than the poor, multiple equilibria may exist, with the rich living either in the center or the suburbs. In effect, the high income (and thus strong endogenous amenities) of their own neighborhood makes the rich reluctant to leave it wherever it might be located. However, the analysis shows that if the center has a strong exogenous-amenity advantage, then the *only* equilibrium has the rich living in the center. Therefore, to generate a location pattern with the rich in the suburbs, the center's exogenous-amenity advantage cannot be too great, as in previous model.

Regardless of whether or not endogenous amenities are present, the theory thus predicts that the pattern of exogenous amenities in a city has an important effect on location by income. But since the pattern of such amenities is highly idiosyncratic, varying across cities depending on their history and topography, the model predicts a variety of location patterns by income across cities. Given the variety of location patterns actually observed around the world, which seem to defy prediction by the standard model, this is a welcome implication.

The last step in the discussion is to consider why the pattern of exogenous amenities differs across cities. A central city's natural amenities are partly governed by transportation needs, which often dictate proximity to a body of water. On the other hand, historical amenities are determined mainly by past government decisions regarding investment in urban infrastructure. Section 4 presents a discussion of these factors, attempting to explain why exogenous amenities are more favorable in European than American central cities. Section 5 offers conclusions.

2. A model with exogenous amenities

To develop a formal model, let x denote distance to the CBD, and let the exogenous amenity level at distance x be given by $a(x)$. Consumer utility depends on amenities as well as on housing consumption, q, and consumption of a numeraire nonhousing good, denoted e, with the utility function given by $u(e, q, a)$. Assuming initially that the city has a single-income group, let income equal y and commuting cost per mile be given by t, so that disposable income at distance x is $y - tx$. The budget constraint is then $e + pq = y - tx$, where p is the price per unit of housing, and elimination of e allows utility to be written as $u(y - tx - pq, q, a)$.

The consumer maximizes this expression by choice of q taking p as parametric, which yields the first-order condition $u^q = pu^e$ (supercripts denote partial derivatives). But p must vary with x to ensure that utility is the same in all locations. Letting $\bar{u}$ denote the uniform utility level,[4] this requirement means

[4] The ensuing analysis applies regardless of whether the city is open or closed (i.e., whether $\bar{u}$ is exogenous or endogenous).

that $\max_{\{q\}} u(y - tx - pq, q, a) = \bar{u}$. Together, this equation and above first-order condition determine q and p as functions of location. These functions are $q(x)$ and $p(x)$, with the latter giving the 'bid-price' function for housing. To find the slope of the bid-price function, the uniform-utility condition is differentiated with respect to x, which yields

$$-[t + p'(x)q(x) + p(x)q'(x)]u^e + q'(x)u^q + a'(x)u^a = 0. \quad (1)$$

Since the $q'(x)$ terms in Eq. (1) cancel given $u^q = pu^e$, rearrangement gives the desired slope:

$$p'(x) = -\frac{t}{q(x)} + \frac{u^a}{q(x)u^e}a'(x) = -\frac{t}{q(x)} + \frac{v^a[y - tx, p(x), a(x)]}{q(x)}a'(x). \quad (2)$$

In the second part of Eq. (2), the marginal rate of substitution u^a/u^e is rewritten as the amenity derivative of the indirect utility function $v[y - tx, p(x), a(x)]$. Note that v^a gives the marginal valuation of amenities *after optimal adjustment of housing consumption.*

The standard urban model ignores amenities by assuming $v^a \equiv 0$. Eq. (2) shows that in this case, the housing price falls with distance to the CBD, compensating suburban consumers for their high commuting costs. This conclusion is reinforced if $a' < 0$, indicating that amenities decline with distance, because suburban housing prices must then compensate for inferior amenities as well as the high cost of commuting. However, if $a' > 0$, then the net advantage of the suburbs is ambiguous, and this is reflected in an ambiguous spatial behavior for p (i.e., Eq. (2) can take either sign). To simplify the discussion, the commuting-cost effect is assumed to dominate in this case, making p' negative at all locations regardless of the pattern of amenities.

Now, let the model be enlarged to include two income groups, poor and rich. Their incomes are y_0 and y_1, respectively, with $y_0 < y_1$, and their commuting cost parameters are t_0 and t_1. Since the value of time is higher for the rich, it follows that $t_1 > t_0$. The above analysis then generates two bid-price functions, one for each group, and these are denoted $p_0(x)$ and $p_1(x)$.

To develop the theory of location by income, observe that a given group will occupy the area of the city where it outbids the other group for housing. The boundary between the areas, denoted $\hat{x}$, is then the point where the groups' bid prices are equal, satisfying $p_0(\hat{x}) = p_1(\hat{x})$. Next, observe that the relative slopes of the bid-price curves at $\hat{x}$ determine where a given group's bid is maximal. If $p_1'(\hat{x}) > p_0'(\hat{x})$, so that the poor curve is steeper (more negatively sloped) at $\hat{x}$, then the poor outbid the rich for central housing while the rich outbid the poor for suburban housing. If $p_1'(\hat{x}) < p_0'(\hat{x})$, so that the rich curve is steeper, then the reverse pattern obtains.

Using Eq. (2), the difference between the bid-price slopes at $\hat{x}$ for the two groups can be written as

$$\Delta \equiv p_1'(\hat{x}) - p_0'(\hat{x}) = \frac{t_0}{q_0(\hat{x})} - \frac{t_1}{q_1(\hat{x})} + a'(\hat{x})\left(\frac{v^a[y_1 - t_1\hat{x}, p_1(\hat{x}), a(\hat{x})]}{q_1(\hat{x})} - \frac{v^a[y_0 - t_0\hat{x}, p_0(\hat{x}), a(\hat{x})]}{q_0(\hat{x})}\right), \quad (3)$$

where $q_0(x)$ and $q_1(x)$ give housing consumption for the two groups. Referring to Eq. (3), the rich locate in the suburbs (center) when $\Delta > (<) 0$.

Using these principles, consider the standard analysis of location by income, which assumes $v^a \equiv 0$. Under this assumption, Eq. (3) yields $\Delta = t_0/q_0(\hat{x}) - t_1/q_1(\hat{x})$. To sign this expression, note that since the price of housing is the same for both groups at $\hat{x}$, the difference between $q_0(\hat{x})$ and $q_1(\hat{x})$ is solely a function of the difference in incomes. Since $y_1 - t_1\hat{x} > y_0 - t_0\hat{x}$ holds despite $t_1 > t_0$, it follows that $q_1(\hat{x}) > q_0(\hat{x})$. As noted in the introduction, this makes comparison of the t/q ratio across groups ambiguous. If t rises *less rapidly* than q as income increases, then $t_0/q_0(\hat{x}) > t_1/q_1(\hat{x})$ holds and $\Delta > 0$, implying that the rich live in the suburbs. If t rises *faster* than q as income increases, $t_0/q_0(\hat{x}) < t_1/q_1(\hat{x})$ holds and $\Delta < 0$, implying that the rich live in the center.

To formalize the main argument of the paper, let the amenity effect be reintroduced. To evaluate the sign of Δ, which now depends on the amenity term in Eq. (3), two assumptions are made. First, conventional locational effects, as embodied in the t/q ratio, are assumed to favor suburban location of the rich. The difference in the t/q ratios in Eq. (3) is then positive. Second, v^a is assumed to rise with income, and its rise is assumed to be more rapid than the increase of housing consumption. Thus, the marginal valuation of amenities (after adjustment of q) rises faster than housing consumption itself. This assumption means that the difference between the v^a/q ratios in Eq. (3) is positive (note that this conclusion uses the fact that the v^a terms have equal price arguments). To show that the assumed behavior of v^a/q is not unreasonable, the appendix presents an example showing that this ratio, indeed, rises with income under CES preferences, provided that a mild parameter restriction holds.

Under these assumptions, consider the effect of the amenity pattern on the sign of Eq. (4). If $a'(\hat{x})$ is negative but small in absolute value, then the entire amenity term in Eq. (4) is similarly negative but close to zero. The positive sign of the first part of Eq. (4) will then dominate, yielding $\Delta > 0$. Thus, if the center's amenity advantage over the suburbs is weak, the U.S. location pattern holds: the poor live in the center and the rich live in the suburbs. The same conclusion holds if the amenity function is upward sloping, so that $a'(\hat{x}) > 0$.

On the other hand, if $a'(\hat{x})$ is negative and large in absolute value, then the amenity term will dominate the conventional forces in determining the sign of

Eq. (4). The sign is then negative, so that $\Delta < 0$. Thus, if the center has a large amenity advantage, so that amenities fall rapidly with distance, then the U.S. pattern is reversed: the rich live in the center and the poor live in the suburbs. As noted above, this corresponds to the case of Paris, which has a steep amenity gradient and central location of the rich.

Thus, under the maintained assumptions, the pattern of location by income *can be reversed by steepening the city's exogenous amenity gradient*. Superior amenities make the central city rich, while weak amenities make it poor. Since location by income is then linked to a city's idiosyncratic features, the multiplicity of observed location patterns around the world becomes explicable.

Unfortunately, the above argument is imprecise. The reason is that the other variables in Eq. (3) are held fixed as the magnitude of $a'(\hat{x})$ is altered, and this is done without proper justification. To supply the required justification, let the analysis be carried out by comparing two cities: a *reference* city and a *comparison* city. The reference city has the location pattern of the first city considered above, where the rich live in the suburbs. Its $\hat{x}$ value is denoted $\hat{x}_r$, and its amenity function is $a_r(x)$. The comparison city satisfies a number of requirements. For each group, income, commuting cost, and utility are the same as in the reference city. In addition, the comparison city's amenity function, denoted $a_c(x)$, has the property that $a_c(\hat{x}_r) = a_r(\hat{x}_r)$. Thus, the two amenity functions intersect at $\hat{x}_r$, the group boundary in the reference city.

This assumption has a number of implications. First, it implies that the poor's bid-price at distance $\hat{x}_r$ in the comparison city equals their bid-price at $\hat{x}_r$ in the reference city.[5] The rich bid-prices are similarly equal at $\hat{x}_r$. Since the rich and poor bid-price curves intersect at $\hat{x}_r$ in the reference city, it follows that the comparison city's curves also intersect at $\hat{x}_r$. This in turn implies that the group boundaries are the same in the two cities (i.e., $\hat{x}_c = \hat{x}_r$). Finally, with prices and amenities equal at the common boundary distance, housing consumption for a given group must be the same at this distance in both cities.

Let the common group boundary in the two cities be denoted $\hat{x}$, and let the common amenity level at this location be denoted $\hat{a}$. In addition, let the common housing price level at $\hat{x}$ be $\hat{p}$, and let the common levels of poor and rich housing consumption be denoted $\hat{q}_0$ and $\hat{q}_1$. The only variable whose value is different at $\hat{x}$ in the two cities is then the *slope* of the amenity function, which equals $a_r'(\hat{x})$ in the reference city and $a_c'(\hat{x})$ in the comparison city. To emphasize this difference,

[5] To see why both cities must have the same poor bid-prices at $\hat{x}_r$, recall that incomes and commuting costs are the same and that amenities at $\hat{x}_r$ are the same by assumption. Thus, for the utilities of the poor to be equal across cities, bid-prices at distance $\hat{x}_r$ must be the same. The same argument applies to the rich.

the Δ expression in Eq. (3) can be written twice, once for the reference city and once for the comparison city. This yields

$$\Delta_r = \frac{t_0}{\hat{q}_0} - \frac{t_1}{\hat{q}_1} + a_r'(\hat{x})\left(\frac{v^a[y_1 - t_1\hat{x}, \hat{p}, \hat{a}]}{\hat{q}_1} - \frac{v^a[y_0 - t_0\hat{x}, \hat{p}, \hat{a}]}{\hat{q}_0}\right), \tag{4}$$

$$\Delta_c = \frac{t_0}{\hat{q}_0} - \frac{t_1}{\hat{q}_1} + a_c'(\hat{x})\left(\frac{v^a[y_1 - t_1\hat{x}, \hat{p}, \hat{a}]}{\hat{q}_1} - \frac{v^a[y_0 - t_0\hat{x}, \hat{p}, \hat{a}]}{\hat{q}_0}\right). \tag{5}$$

Under this formulation, the procedure of holding the other elements of the Δ expression fixed as the amenity gradient changes is proper. The key constraint is that the amenity function continues to intersect the original function at the fixed $\hat{x}$ as it steepens, as shown in Fig. 1. As above, such steepening ultimately generates a reversal of the U.S. location pattern. This conclusion is summarized as follows:

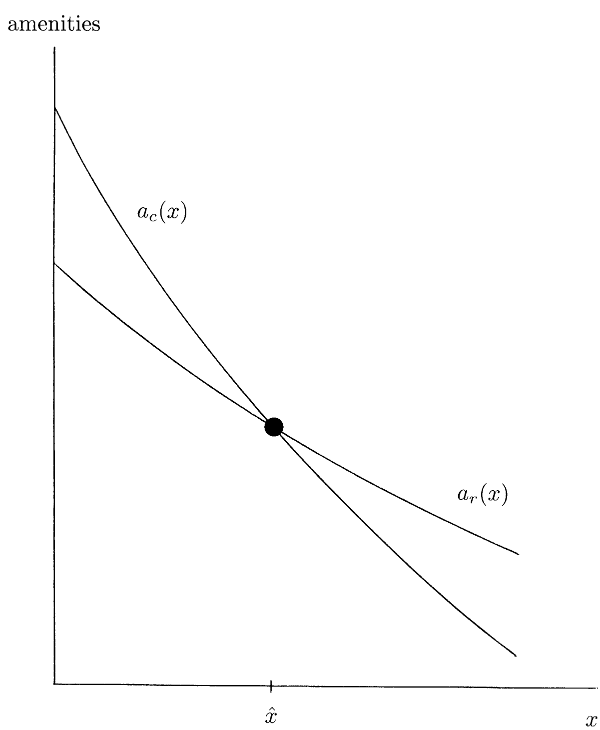

Fig. 1. Amenity functions.

Proposition. Suppose that the conventional locational forces dominate in the reference city, so that $\Delta_r > 0$ holds and the rich live in the suburbs. Suppose further that the marginal amenity valuation v^a rises faster than housing consumption as income increases. Then, the location pattern will be reversed in the comparison city, with $\Delta_c < 0$ holding and the rich living in the center, whenever $a_c'(\hat{x})$ is negative and sufficiently large in absolute value.

3. A model with endogenous amenities

The analysis up to this point has assumed that amenities are exogenous. However, as explained in the introduction, the modern amenities available at a given location depend in part on the local income level. Such amenities are therefore a by-product rather than a determinant of the location patterns of different income groups. In this section, the analysis is broadened to include endogenous amenities. The modified model involves a number of simplifying assumptions and is meant only to be suggestive.

The utility function is now written $u(e, q, a, z)$, where a again represents exogenous amenities and z represents the income of the consumer's neighborhood. Modern amenities are assumed to be an increasing function of neighborhood income. To simplify the characterization of neighborhoods, the continuous distance measure is dropped and replaced by a discrete measure that recognizes just two locations: center and suburbs. The z value in a given location then depends on which income group lives there. Commuting costs at the center are equal to zero, while total commuting costs from the suburbs equal t_0 and t_1 for the poor and rich (suburban distance is normalized to one).

For simplicity, it is also assumed that dwellings at any location are available in two fixed sizes, q_0 and $q_1 > q_0$, with the rich choosing the larger size. Finally, the poor are assumed to be indifferent to the level of exogenous amenities and neighborhood income, while the rich value both. This assumption, which mirrors the one made in Section 2, means that $u^a = u^z = 0$ holds when e and q are small while $u^a, u^z > 0$ holds when e and q are large.

Let $\tilde{p}_0$ denote the poor's bid-price for housing in the center and $\bar{p}_0$ denote their bid-price in the suburbs. Since these prices must equalize the utility of the poor in the two locations, it follows that

$$u(y_0 - \tilde{p}_0 q_0, q_0, \tilde{a}, \tilde{z}) = u(y_0 - t_0 - \bar{p}_0 q_0, q_0, \bar{a}, \bar{z}), \tag{6}$$

where $\tilde{a}$ and $\tilde{z}$ give exogenous amenities and neighborhood income in the center and $\bar{a}$ and $\bar{z}$ give the suburban values. Because the poor are unaffected by a and z, the difference in these variables between center and suburbs has no effect on the bid prices in Eq. (6). Satisfaction of Eq. (6) requires only that nonhousing consumption be equal in the two locations, which implies $\tilde{p}_0 q_0 = t_0 + \bar{p}_0 q_0$.

Rearranging, the bid-price differential for the poor between center and suburbs then equals

$$\tilde{p}_0 - \bar{p}_0 = \frac{t_0}{q_0}. \tag{7}$$

Since the rich care about neighborhood income, their bid prices at the different locations depend on the residential pattern. To see this, let the pattern which has the rich in the center and the poor in the suburbs be denoted A, with B denoting the reverse pattern. The rich bid-prices under pattern A, denoted $\tilde{p}_1^A$ and $\bar{p}_1^A$, must then satisfy

$$u(y_1 - \tilde{p}_1^A q_1, q_1, \tilde{a}, y_1) = u(y_1 - t_1 - \bar{p}_1^A q_1, q_1, \bar{a}, y_0). \tag{8}$$

Note that under pattern A, $\tilde{z} = y_1$ and $\bar{z} = y_0$. Observe also that $\bar{p}_1^A$ gives the price that a rich person would pay to live amidst the poor in the suburbs, a movement that would upset the assumed residential pattern. Assuming that $\tilde{a} > \bar{a}$, the center has better exogenous amenities than the suburbs and offers better modern amenities, a consequence of its higher income. Therefore, utility equalization for the rich requires *lower* nonhousing consumption in the center, which means $\tilde{p}_1^A q_1 > t_1 + \bar{p}_1^A q_1$. Rearranging, the rich's bid-price differential between suburbs and center under pattern A satisfies

$$\tilde{p}_1^A - \bar{p}_1^A > \frac{t_1}{q_1}. \tag{9}$$

As in the model with continuous distance, the group with the larger bid-price differential lives in the center.[6] Using this principle, the bid-price differentials for the rich and poor must be compared to decide whether pattern A is an equilibrium. Assuming $t_0/q_0 > t_1/q_1$ as before, a comparison of Eqs. (7) and (9) shows that pattern A may, indeed, satisfy the requirements of equilibrium. In other words, $\tilde{p}_1^A - \bar{p}_1^A > t_0/q_0 = \tilde{p}_0 - \bar{p}_0$ may hold, indicating that the rich have the larger bid-price differential, provided that $\tilde{p}_1^A - \bar{p}_1^A$ is much larger than t_1/q_1. In this case, $\tilde{p}_1^A - \bar{p}_1^A$ will exceed t_0/q_0 even though $t_0/q_0 > t_1/q_1$. For this outcome to occur, the center's exogenous amenity advantage must be sufficiently large.

The intuitive explanation follows the argument of Section 2. A rich person moving to the suburbs would forsake exogenous amenities and would also enter a lower-income area, with a consequent loss of modern amenities. If center has

[6] In equilibrium, the value of p in the suburbs would equal a constant δ that depends on the opportunity cost of urban land. The p value in the center would then equal δ plus the *smaller* of the bid-price differentials. Because the group living in the center has the larger differential, its bid for suburban housing would then be less than δ, ensuring that the residential pattern is an equilibrium.

a large amenity advantage, these losses will overwhelm the conventional forces that draw the rich to the suburbs, assuring that pattern A is an equilibrium.

Observe that if the endogenous, modern amenities are dropped from the model, the above argument is unaffected and the conclusion mirrors that of Section 2. More noteworthy is the fact that pattern A can be an equilibrium when exogenous amenities are *absent*, with the endogeneous component operating by itself. In this case, central location of the rich generates an endogenous amenity in the center that is sufficient to maintain the location pattern despite the lure of cheaper housing in the suburbs (and the absence of exogenous amenities).

Pattern B, where the rich locate in the suburbs, can also be an equilibrium with endogenous amenities. The bid-prices for the rich under this pattern, denoted $\tilde{p}_1^B$ and $\bar{p}_1^B$, must satisfy

$$u(y_1 - \tilde{p}_1^B q_1, q_1, \tilde{a}, y_0) = u(y_1 - t_1 - \bar{p}_1^B q_1, q_1, \bar{a}, y_1). \tag{10}$$

Since $\tilde{a} > \bar{a}$ while modern amenities are worse in the center, the center's net advantage over the suburbs is ambiguous. As a result, nonhousing consumption could be either smaller or larger in the center, implying that the bid-price differential $\tilde{p}_1^B - \bar{p}_1^B$ could be larger or smaller than t_1/q_1. However, as long as $\tilde{p}_1^B - \bar{p}_1^B$ is less than the poor differential t_0/q_0, pattern B is an equilibrium. Note that this outcome is assured if exogenous amenities are absent, which implies $\tilde{p}_1^B - \bar{p}_1^B < t_1/q_1 < t_0/q_0$.

This discussion shows that endogenous amenities introduce the possibility of multiple equilibria. In particular, patterns A and B are both equilibria when

$$\tilde{p}_1^A - \bar{p}_1^A > t_0/q_0 > \tilde{p}_1^B - \bar{p}_1^B. \tag{11}$$

For the first inequality in Eq. (11) to be satisfied, $\tilde{a}$ must be large enough relative to $\bar{a}$ so that the combined exogenous and endogenous amenity advantage of the center under pattern A is large enough to offset the pull of the conventional forces, keeping the rich in the center. For the second inequality in Eq. (11) to hold, $\tilde{a}$ must not be so large as to offset the combined pull of the conventional forces and endogenous suburban amenities under pattern B, keeping the rich in the suburbs.

The intuitive reason for the existence of multiple equilibria is that endogenous amenities make the existing location of the rich attractive to them wherever it might be. Moving into the poor area requires a sacrifice of endogenous amenities, which depresses the bid-price of the rich and tends to maintain the existing location pattern, whether it is A or B. Note that the possibility of multiple equilibria may help explain the puzzling variety of location patterns by income. However, this explanation is not as compelling as in the exogenous-amenities case because it requires one to argue that different equilibria are somehow selected in different cities.

Despite this ambiguity, the clearcut implications of the Section 2's analysis, namely that strong exogenous amenities in the center ensure central location of the rich, also emerges in the presence of endogenous amenities. This follows because both bid-price differentials in Eq. (11) are increasing in $\tilde{a}$, the level of central amenities. As a result, once $\tilde{a}$ becomes sufficiently large, the second inequality in Eq. (11) is violated, ruling out pattern B as an equilibrium. Thus, as in Section 2, the rich must live in the center whenever its exogenous amenity advantage is sufficiently large.

4. Why exogenous-amenity patterns differ across cities

Because the theory says that location by income depends on the spatial pattern of exogenous amenities, it is important to ask why such amenity patterns differ across cities. To start, consider the pattern of natural amenities, focusing on water access. Since the need for cheap transportation often caused urban areas to develop on seacoasts, rivers or lakes, many European and U.S. central cities enjoy the natural amenity of proximity to a body of water. Given this common outcome, differential access to water appears not to be an important source of amenity differences between European and U.S. central cities. In the case of Paris, however, the Seine river seems to play a larger role in the life of the city than do water-related amenities in many other urban areas. As a result, this particular natural amenity may be an important factor in generating central Paris's advantage over its suburbs.

Turning to historical amenities, Europe's longer history provides an obvious reason why its central cities contain more buildings and monuments of historical significance than do their U.S. counterparts. Many European cities were major metropolises at a time when much of the U.S. had not even been settled, and the legacy of urban development from this distant past provides an atmosphere in European city centers that appears to be highly valued by the residents.

In addition to the effect of a longer history, government investment in central-city infrastructure appears in many cases to have been more extensive in European cities than in the U.S. This in turn may reflect different political arrangements, as argued in the literature on urban 'primacy.' This literature, which attempts to explain why many countries contain a single, dominant city, argues that an important factor is a highly-centralized national government. When a country has such a government, tax resources are heavily invested in developing the capital city, which then tends to become dominant (see Henderson, 1988). Ades and Glaeser (1995) offer a similar argument, claiming that when the residents of given city amass political power (as when the city is the capital of a centralized country), government resources flow toward that city, making it large. Even though these arguments are directed toward explaining primacy in a centralized nation, they also imply that lavish government investment in buildings, monuments, parks, and other infrastructure is likely to occur over the capital city's history in such a country, generating a foundation for the historical amenities enjoyed by present-day residents.

Being the capital city of country with a strong central government, Paris clearly fits the above description. In addition, it is apparent that many of the historical amenities in central Paris are the result of substantial investment of national tax revenues over the city's history, as the above argument would predict. The same phenomenon is obviously at work in other European cities such as London, whose abundance of historical amenities can be traced in part to a dominant role in the life of its country.[7] By contrast, the greater degree of political decentralization in the U.S. means that the capital city, Washington, DC, benefits less from national tax revenue than would otherwise be the case. Indeed, the fiscal crisis currently underway in the District of Columbia would be unimaginable in a country like France or Britain. As a result, although Washington offers historical amenities, their effect is not strong enough to generate a Parisian-style pattern of location by income.

Although the previous discussion might suggest otherwise, historical amenities actually depreciate over time, which means that their maintenance requires ongoing investment. If such expenditures were withheld, the central city's amenities would decay, and high-income residents would be increasingly drawn to the suburbs. In the case of Paris, it is obvious to any visitor that substantial expenditures of this type are devoted to maintaining the stock of historical amenities.[8]

One could argue that such expenditures arise for the same political reasons as the original investment in historical infrastructure, namely the dominance of the capital city. In addition, in the case of Paris, it is clear that spending on amenity restoration and maintenance is also designed to preserve the city's appeal for tourism, which generates large benefits for the local economy. But, as noted in the introduction, such spending also has the features of an endogenous, modern amenity that responds to the local income level in the same way as the number of restaurants or theaters. In other words, amenity maintenance is a local public good whose allocation may be skewed toward politically important,

[7] In Brussels, a substantial part of the historical center has been demolished and replaced by office buildings, a phenomenon that has been dubbed 'Brusselization' by disapproving Europeans. Moreover, location by income in that city follows the U.S. pattern, which is consistent with the present theory (see Thomas and Zenou, 1997, for evidence).

[8] One highly visible expenditure supports the fabled corps of Paris street cleaners, who maintain an immaculate central-city environment. According to Nicot (1996), other French urban areas have stimulated the movement of high-income households to their central cities by following Paris' example, using tax revenues to restore historical districts.

high-income neighborhoods.[9] This argument predicts that if the rich were somehow to relocate to the suburbs, the central city's historical amenities might wither through lack of maintenance.

This view, which suggests that historical amenities are themselves partly endogenous, may contain an element of truth. Nevertheless, the historical amenities in European central cities are best viewed as exogenously determined. Their presence, which arises from a long European history and the lavish past investments of strong central governments, constitutes an important difference between the U.S. and European cities. This difference may in turn help explain the different patterns of location by income in the two cases.

5. Conclusion

Despite substantial progress in urban economic theory since the 1960s, the absence of a convincing and robust explanation of location by income represents a significant failure of the standard model. This paper has attempted to remedy that failure by presenting an amenity-based theory. The theory demonstrates that the relative location of different income groups depends on the spatial pattern of exogenous amenities in a city. The analysis shows that when the center has a strong advantage over the suburbs in exogenous amenities, and when valuation of these amenities rises rapidly with income, the rich are likely to live at central locations. This conclusion applies regardless of whether or not endogenous amenities, which depend an area's income level, are present. The virtue of the theory is that it ties location by income to a city's idiosyncratic characteristics. It thus predicts a multiplicity of location patterns across cities, consistent with real-world observation.

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Appendix A

The argument in Section 2 hinges on the assumption that v^a/q is increasing in income, and it is important to know whether this assumption is consistent with

[9] Empirical evidence is scarce on the extent to which government spending within jurisdictions is skewed in favor of high-income residents. For one study, see Behrman and Craig (1987), who investigate the allocation of police across neighborhoods in Baltimore.

familiar specifications of preferences. The following discussion shows that the assumption is, in fact, consistent with CES preferences under a particular parameter restriction.

Suppose the utility function takes the CES form

$$u(e, q, a) \equiv [\alpha e^{-\theta} + \beta q^{-\theta} + (1 - \alpha - \beta) a^{-\theta}]^{-1/\theta}. \tag{A.1}$$

Differentiation then establishes

$$\frac{u^a}{u^e} = \frac{(1 - \alpha - \beta)}{\alpha} \left(\frac{e}{a}\right)^{1/\sigma}, \tag{A.2}$$

where $\sigma = 1/(1 + \theta)$ is the elasticity of substitution. The demand functions for e and q are given by $e(x) = (y - tx)\,[\Omega(x) / (\Omega(x) + p(x))]$ and $q(x) = (y - tx) / (\Omega(x) + p(x))$, where $\Omega(x) = (\alpha p(x)/\beta)^{\sigma}$. Substituting the expression for $e(x)$ into Eq. (A.2), and then dividing the result by the expression for $q(x)$ yields

$$\frac{v^a[y - tx, p(x), a(x)]}{q(x)} = \frac{(1 - \alpha - \beta)}{\beta} p(x)\, a(x)^{-1/\sigma} (y - tx)^{(1-\sigma)/\sigma} [\Omega(x) + p(x)]^{-(1-\sigma)/\sigma}. \tag{A.3}$$

Inspection of Eq. (A.3) shows that v^a/q is increasing in income, as assumed in the above proposition, provided that the elasticity of substitution is less than one (the exponent of $y - tx$ is then positive). A low elasticity of substitution means that the indifference curves are strongly bowed toward the origin, and this property in turn implies that their absolute slope $|\partial e/\partial a|$ in (a, e) space increases rapidly as e rises in response to an increase in income. When $\sigma > 1$, the absolute slope rises faster than q, so that v^a/q increases. Thus, the key assumption of the theory presented in Section 2 is validated under a common specification of preferences.[10]

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[10] Note that with Cobb–Douglas preferences, $\sigma = 1$ and v^a/q is independent of income.

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