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MATCHING AND AGGLOMERATION ECONOMIES IN A SYSTEM OF CITIES*

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This paper examines resource allocation in a system of cities with heterogeneous workers and firms and imperfect information. We derive an agglomeration economy in the labor market from a matching process between workers and firms, and show that it has the characteristics of a local public good. We illustrate two externalities associated with firm location, and show that they render free entry equilibria inefficient. We analyze the formation of equilibrium cities as a game, and argue that since profit maximizing land developers cannot control the number of firms directly, they cannot attain efficient city sizes.

1. Introduction

Most theoretical models of agglomeration are based on external scale economies [Chipman (1970)] and assume that the production function of a firm shifts out as city or industry size increases [Moomaw (1981)]. This specification has led to insights into the effects of agglomeration economies on resource allocation [Arnott (1979), Henderson (1974, 1985, 1988)] and evidence about their importance [Henderson (1986)]. External scale economies in cities are attributed to positive externalities that arise when firms locate near other firms. Henderson (1986, p. 48) lists four factors that seem to capture the nature of these external effects:

> '(i) economies of intraindustry specialization where greater industry size permits greater specialization among firms in their detailed functions, (ii) labor market economies where industry size reduces search costs for firms looking for workers with specific training relevant to that industry, (iii) scale for "communication" among firms affecting the speed of, say, adoption of new innovations, and (iv) scale in providing (unmeasured) public intermediate inputs tailored to the technical needs of a particular industry.'

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Although some authors [e.g., Goldstein and Gronberg (1984)] have examined the sources of agglomeration economies, formal models of their microfoundations are rare.

This paper derives an agglomeration economy from optimizing behavior of workers and firms. We extend the standard monocentric model of a residential land market to include a labor market with heterogeneous workers and firms and imperfect information. Each agent chooses a city knowing the number but not the characteristics of the other agents. Both land rents and the profits of firms are redistributed. Workers and firms expect to be matched in a way that maximizes their productivity, and we show that the expected quality of this match rises with city size. Thus, the model formalizes one commonly cited source of agglomeration economies: the ability of large labor markets to provide a better match between jobs and skills.

We use this model to analyze the characteristics of optimum and equilibrium cities. To do this we solve several optimization problems. The first-best optimum maximizes the utility of a representative worker in a typical city. The second-best optimum maximizes utility subject to the requirement that firms expect zero profits. Equilibrium cities arise from a game of city formation in which, as in Henderson (1985), competitive land developers attempt to profit by correcting inefficiencies in cities.

Our analysis yields three insights into the effects of agglomeration economies on resource allocation. First, the agglomeration economy has the characteristics of a local public good. A firm entering a city improves the expected quality of the match between job requirements and skills for all workers, leading to a positive ex ante relationship between wages, productivity, and city size. No worker can be excluded from the benefits of a better expected match; such benefits are also non-rival. As a result, a version of the Henry George theorem [Stiglitz (1977) and the references therein] holds: a confiscatory land tax in an optimum city is sufficient to pay for the provision of an optimal amount of the agglomeration economy.

Second, there are two externalities associated with firm location in our model. One is the conventional productivity externality. If a firm enters a city, it improves the productivity of all workers, but it considers only its own profits. This productivity externality causes a city to contain too few firms under free entry. The other externality arises from spatial competition and the heterogeneity of workers and firms. An entrant to a city reduces the labor market areas, and hence profits, of incumbent firms, but considers only its own profits. This competition externality leads to too many firms under free entry. The competition externality dominates in our model. As a result, cities contain too many firms for a given number of workers under free entry.

Third, equilibrium city sizes are not optimal. We assume that land developers are unable to control the number of firms directly. They can only

offer incentive compatible contracts to workers to induce them to migrate to newly formed cities. This is an important distinction between the feasible actions of land developers and local governments.[1] Thus, free entry leads to the zero profit number of firms in any city in equilibrium. However, because of the externalities discussed above, the zero profit number of firms is not efficient.

Like all models of systems of cities, ours builds on the work of Henderson (1974, 1985, 1988). Our conception of the matching process is related to Arnott's (1988) work on matching in thin housing markets. A primitive difference between his model and ours is that in Arnott's households arrive on the market according to an exogenous Poisson process, while in ours population is endogenously determined by migration. Kim (1987) analyzes a partial equilibrium model of an urban labor market with heterogeneous firms and workers, with emphasis on bargaining and the determination of factor rewards. Unlike Kim, we place labor market matching in a general equilibrium setting where agents have imperfect information. Other recent studies of heterogeneity in cities include Abdel-Rahman (1988), Hobson (1987) and Fujita (1988) on optimum product variety, Helsley and Strange (1988) on urban capital markets, and Schulz and Stahl (1988) on search economies in retailing.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes the expected utility of a worker and the expected profit of a firm in cities of different sizes. Section 4 solves for the first- and second-best allocations of firms and workers to cities. Section 5 shows that the second-best optimum is the equilibrium of a simple city formation game. Section 6 compares the equilibrium and optimum cities. Section 7 summarizes.

2. The model

This is a model of a system of cities with an explicit agglomeration economy in the labor market. There are three sets of agents in the model: (1) perfectly mobile, heterogeneous workers who migrate between cities in response to differences in expected utility; (2) heterogeneous firms who arise in cities to eliminate excess profits; and (3) profit maximizing land developers who encourage the formation of efficient city sizes. All agents have imperfect information. For example, before choosing a city, workers do not know the job requirements of firms and firms do not know the skills of workers. The agglomeration economy arises because workers and firms expect to be better matched in large cities.

This section describes the basic elements of the model: the characteristics

[1]Stiglitz (1977) and Bewley (1981) discuss the importance of active local governments for the attainment of efficient club sizes.

of workers; the market for land; the characteristics of firms; the job matching problem; and the bargaining process that determines wages and profits.

2.1. Workers

There are N workers in the economy. Let n denote the number of workers in a city. Workers have heterogeneous skills, described by an address y on the unit circle. Points on the unit circle constitute the universe of possible job characteristics; the address of a worker indicates the job for which he is best suited. As discussed in detail below, a worker's wage depends on how closely his skill matches his job. Workers may work for one firm only, live and work in the same city, are perfectly mobile, and choose cities to maximize expected utility.

Before selecting a city workers cannot observe the job requirements of firms. Workers know the number of firms and workers in each city, and they form expectations about their wage assuming that the job requirements of firms are random draws from the uniform distribution on the unit circle. In addition, workers know how firms interact, and can infer how they are spaced on the unit circle in equilibrium.

2.2. The land market

Land is not scarce in the aggregate; there are D potential city sites, where D is large enough that none of our solutions are constrained by it. The aggregate population is endogenously partitioned into $K<D$ cities, indexed $k=1,2,\ldots,K$. The determination of the number of cities is made clear in the sequel. We adopt a simple monocentric model of the land market in each city. In any one city, employment is concentrated at a point central business district (CBD) to which workers commute daily. Workers live in a circular residential district centered at the CBD. Each worker occupies one unit of homogeneous land. Locations are completely described by their distance z from the CBD. 2ϕ radians of land are suitable for residential use. The opportunity cost of land is zero.

Workers are risk neutral and have identical utility functions. The utility function of a representative worker is

$$U=w+R+S-r(z)-tz, \tag{2.1}$$

where w is (uncertain) labor income obtained from the sale of one unit of labor, R is income derived from land ownership, S is (uncertain) income derived from a worker's share of the profits of firms, $r(z)$ is land rent at distance z, and t is the cost of commuting a unit of distance.

Three conditions characterize an equilibrium in the land market. First, expected utility is equal at all locations: $\partial V/\partial z = 0$, where

$$V = \mathrm{E}[w] + R + \mathrm{E}[S] - r(z) - tz. \tag{2.2}$$

This implies $r'(z) = -t$, or $r(z) = r(0) - tz$, where $r(0)$ is rent at the CBD. Second, land is allocated to the highest bidder. This implies that urban land rent equals the opportunity cost of land at the boundary of the city: $r(z^*) = 0$, where z^* denotes the location of the boundary. This in turn implies $r(0) = tz^*$ and hence

$$r(z) = t(z^* - z). \tag{2.3}$$

Third, excess demand equals zero. The aggregate demand for land equals n. The aggregate supply of land equals ϕz^{*2}. The market clearing condition $n - \phi z^{*2} = 0$ implies

$$z^* = (n/\phi)^{1/2}. \tag{2.4}$$

Workers receive equal shares of a fraction δ of aggregate differential land rent. In general, δ is chosen by profit maximizing land developers. It is one component of a contract developers offer workers to induce them to migrate to newly formed cities. Rental income is given by $R = (\delta/n)DLR$, where

$$DLR = 2t\phi \int_0^{z^*} (z^* - \mu)\mu \, \mathrm{d}\mu = (t\phi/3)z^{*3} = (t/3)n^{3/2}\phi^{-1/2}, \tag{2.5}$$

and we have substituted for z^* from (2.4). Using (2.5), rental income may be written

$$R = (\delta t/3)(n/\phi)^{1/2}. \tag{2.6}$$

To simplify the exposition, we assume that all land rent is redistributed: $\delta = 1$. This will not affect the calculation of the optimum since we employ an additive social welfare function. In section 5 we show that $\delta = 1$ is a characteristic of equilibrium.

By combining (2.2)–(2.4) with (2.6) and the assumption that $\delta = 1$, expected utility may be written

$$V = \mathrm{E}[w] + \mathrm{E}[S] - (2t/3)(n/\phi)^{1/2}. \tag{2.7}$$

The third term in (2.7) equals income from land ownership minus the sum of land rent and transportation costs. It also equals aggregate transportation cost per worker, $(1/n)ATC$, where

$$ATC = 2t\phi \int_0^{z^*} \mu^2 \, \mathrm{d}\mu = (2t\phi/3)z^{*3} = (2t/3)n^{3/2}\phi^{-1/2}. \tag{2.8}$$

2.3. Firms and job matching

There are M potential firms in the economy; m denotes the number of firms in a city. Entrepreneurial skill is not scarce in the aggregate; $M > mK$. Firms produce a single output q, for which demand is perfectly elastic. We normalize the price of output to one. Firms have heterogeneous job requirements, described by an address x on the unit circle. The address of a firm indicates the type of worker it can employ most efficiently. Let $\alpha > 0$ be the constant productivity of a worker whose skills exactly match his job, and $\beta > 0$ be the loss per unit distance in the characteristic space caused by mismatch. Alternatively, β may be interpreted as the cost (per unit distance) of training a worker to perform a job which requires skills different than his own. The output of the match (x, y) is written

$$\alpha - \beta|x - y|. \tag{2.9}$$

Firms may hire more than one worker. Let $Y(x)$ denote the set of addresses of the workers employed by a firm with job requirement x. The number of workers employed by the firm is denoted by $\Omega(x)$. Using this notation, the technology of a firm with job requirement x may be written

$$q(x, Y) = \alpha\Omega(x) - \beta \sum_{Y(x)} |x - y|. \tag{2.10}$$

The associated cost function is

$$\theta(C, w, x, Y) = C + \sum_{Y(x)} w(x, y), \tag{2.11}$$

where C is the fixed cost of production, and $w(x, y)$ is the wage paid to the worker with skill y.

Firms cannot observe the skills of workers before entering a particular city. Firms know the number and addresses of firms in each city, and the number of workers in each city. They form expectations about their profits

assuming that the skills of workers are random draws from the uniform distribution on the unit circle.[2]

2.4. Wage and profit determination

The value of output is split between wages and profits through a process of bilateral bargaining between a worker and the nearest firm in which the worker and the firm alternate wage offers until one is accepted. Rubinstein (1982) shows that the unique perfect equilibrium to this game when the parties have equal bargaining costs and the time taken to formulate offers is arbitrarily small is an immediate agreement to split the value of output.[3] Of course, there are other ways to formulate the determination of wages and profits.[4] However, none are more compelling than 'split the difference'.

The wage is

$$w(x, y) = (1/2)[\alpha - \beta|x - y|]. \tag{2.12}$$

Profits are then given by

$$\begin{aligned}\pi(C, x, Y) &= q(x, Y) - \theta(C, w, x, Y)\\ &= (1/2)\left[\alpha\Omega(x) - \beta \sum_{Y(x)} |x - y|\right] - C.\end{aligned} \tag{2.13}$$

3. Expected profits and expected utility

3.1. Expected profits

Each firm expects to employ the workers whose skills lie in its market area, the set of points on the unit circle that are closer to this firm than to any other. This expectation is rational in the sense that it is consistent with expected utility maximization for workers and expected profit maximization for firms. Firms choose a location in the characteristic space to maximize expected profits. Since firms are identical ex ante, the market area of firms are equal in equilibrium. The market area of a firm with job requirement x is $(x - 1/(2m), x + 1/(2m))$.

The number of workers in a firm's market area, $\Omega(x)$, is a binomial

[2]Although there is no process that generates uncertainty in our model, it is easy to conceive of one. In a dynamic context, uncertainty could arise from changes in the characteristics of workers or the technologies of firms.

[3]Shaked and Sutton (1984) give a very simple proof of this result.

[4]Kim (1987) discusses wage determination in a related model in which a worker may bargain with more than one firm.

random variable with parameters n and $1/m$.[5] Hence, expected employment is given by

$$\mathrm{E}[\Omega] = n/m. \tag{3.1}$$

The expected distance between workers and firms, or the expected value of $|x-y|$ conditioned on $y \in (x-1/(2m), x+1/(2m))$, is given by

$$\mathrm{E}[|x-y| : y \in (x-1/(2m), x+1/(2m))] = 1/(4m). \tag{3.2}$$

Note that the expected quality of the match between job requirements and skills improves as the number of firms increases. From (3.1) and (3.2), the expected aggregate distance between the firm and its employees is

$$\mathrm{E}\left[\sum_{Y(x)} |x-y|\right] = (n/m)(1/4m). \tag{3.3}$$

Finally, from (2.13), (3.1) and (3.3), expected profit is given by

$$P(n,m) = (1/2)(n/m)[\alpha - \beta/(4m)] - C. \tag{3.4}$$

$P(n,m)$ is increasing in n, since expected employment increases with the number of workers in a city. Increasing the number of firms has two opposing effects on expected profits. First, as the number of firms increases, the expected quality of the match between skills and job requirements improves, which tends to increase expected profits. Second, competition for workers intensifies as the number of firms increases, decreasing expected employment and expected profits. Since the two effects work in opposite directions, the relationship between expected profits and the number of firms is ambiguous. Under the assumption that all possible matches are productive ($\alpha > \beta/2$), the latter effect dominates, and expected profits decline as the number of firms rises.

3.2. Expected utility

Each worker expects to be employed by the firm whose job requirement best matches his skill. Like the expectations of firms, this is consistent with the maximization of expected utility and profit. Knowing the number of firms and that they are identical, workers infer that market areas equal $1/m$. Following the construction of (3.2), conditioned on being in any firm's

[5]Appendix A explains the derivations of (3.1) and (3.2) in detail.

market area, workers expect $|x-y|$ to equal $1/(4m)$. Thus, from (2.12), the expected wage is

$$\mathrm{E}[w]=(1/2)[\alpha-\beta/(4m)]. \tag{3.5}$$

Note the explicit agglomeration economy in the labor market: as the number of firms increases, the quality of the match between skills and job requirements improves, which increases productivity and wages.

Assuming that workers own equal shares of firms in their city,

$$\mathrm{E}[S]=(m/n)P(n,m). \tag{3.6}$$

Then, from (2.7) and (3.4)–(3.6), expected utility is

$$V(n,m)=\alpha-\beta/(4m)-(m/n)C-(2t/3)(n/\phi)^{1/2}. \tag{3.7}$$

4. Optimum cities

4.1. First-best optimum

In the first-best optimum a planner chooses the number of workers and firms in a city to maximize a Benthamite social welfare function. The planner has imperfect information: he does not know the skills of workers or the job requirements of firms. In the next section, we examine the second-best optimum in which the planner chooses the number of workers, but the number of firms is determined by free entry.

The planner's objective is to maximize aggregate expected utility subject to an aggregate population constraint:

$$\max_{n_k,m_k,K}\sum_{k=1}^{K} n_k V(n_k,m_k) \quad \text{subject to} \quad \sum_{k=1}^{K} n_k=N. \tag{4.1}$$

It is well known that maximum social welfare may involve unequal city sizes when the aggregate population is not an integer multiple of the utility maximizing population of an individual city. Wooders (1978) shows that this integer problem is insignificant in sufficiently large economies. Following Wooders, we assume aggregate population is large enough that the integer problem and its attendant non-convexities are unimportant. Thus we are spared the non-existence and instability problems discussed in Pauly (1970) and Stiglitz (1977). This assumption, together with our earlier assumptions of arbitrarily many identical city sites, and identical workers and firms ex ante, implies that cities are the same size, and offer the same level of utility at the

optimum. Thus, $V(n_k, m_k) = V(n, m)$ for all k, which in turn implies $K = N/n$. Then, (4.1) becomes

$$\max_{n,m} NV(n, m). \tag{4.2}$$

With the total population, N, fixed, maximizing aggregate utility is equivalent to maximizing per capita utility in a representative city.

From (3.7), the optimization program is

$$\max_{n,m} V(n, m) = \alpha - \beta/(4m) - (m/n)C - (2t/3)(n/\phi)^{1/2}. \tag{4.3}$$

The first-order conditions are:

$$(m/n^2)C - (t/3)(n\phi)^{-1/2} = 0, \tag{4.4}$$

$$\beta/(4m^2) - C/n = 0. \tag{4.5}$$

Eq. (4.4) characterizes the optimum number of workers in a city. It says that adding another worker at the optimum decreases fixed cost per worker by the same amount that it increases aggregate transportation cost per worker. Eq. (4.5) characterizes the optimum number of firms in a city. It says that adding another firm at the optimum increases output per worker by the same amount that it increases fixed costs per worker.[6]

Eq. (4.4) has another interesting interpretation. Multiplying by n^2 and rearranging terms yields

$$mC = (t/3)\phi^{-1/2}n^{3/2}. \tag{4.6}$$

The left side of (4.6) equals aggregate fixed costs (*AFC*). From (2.5), the right side of (4.6) equals aggregate differential land rents (*DLR*). Hence, aggregate fixed costs equals aggregate differential land rents at the first-best optimum. This is an instance of the Henry George theorem [Stiglitz (1977), Arnott and Stiglitz (1979)] on the optimal size of an economy with local public goods. In local public finance, the theorem states that expenditure on public goods equals aggregate differential land rents at the optimum. Hence, a non-distortionary tax that confiscates all land rent generates just enough revenue to provide an efficient level of the public good to a community of optimal size. Similarly, (4.6) implies that in our model a tax that confiscates all land rent generates just enough revenue to cover the fixed costs of the efficient

[6]The second-order condition is satisfied at the optimum, since $V_{mm}V_{nn} - V_{nm}^2 = (t\beta/6)\phi^{-1/2}n^{-3/2}m^{-3} > 0$.

number of firms when population is set optimally. Further, the analogy is even more precise. In our model, every firm is providing a public good: each one contributes to the expected quality of the match between workers and firms. If another firm locates in the city, wages for all workers are expected to rise. This benefit is external to the firm, and is both non-excludable and non-rival. In short, it is a local public good. The aggregate cost of providing an efficient level of this public good is mC at the optimum; (4.6) states that this cost can be covered by a confiscatory tax on land.

4.2. The second-best optimum

In the second-best optimum the number of firms is determined by free entry. As discussed in the next section, the second-best optimum is an equilibrium under certain assumptions.

Free entry leads firms to expect to earn zero profits. Thus, the optimization program is $\max_{n,m} V(n,m)$ subject to $P(n,m)=0$. From (3.7) and (3.4), the Lagrangean for this problem is

$$L=\alpha-\beta/(4m)-(m/n)C-(2t/3)(n/\phi)^{1/2}$$
$$+\lambda[C-(1/2)(n/m)(\alpha-\beta/(4m))]. \tag{4.7}$$

The first-order conditions are:[7]

$$\partial L/\partial n=(m/n^2)C-(t/3)(n\phi)^{-1/2}-\lambda(1/2m)(\alpha-\beta/(4m))=0, \tag{4.8}$$

$$\partial L/\partial m=\beta/(4m^2)-C/n+\lambda(n/(2m^2))(\alpha-\beta/(2m))=0, \tag{4.9}$$

$$\partial L/\partial\lambda=C-(1/2)(n/m)(\alpha-\beta/(4m))=0. \tag{4.10}$$

λ is the shadow price of fixed costs, the rate at which maximum expected utility per worker changes as fixed costs increase. From (4.9) and (4.10),

$$\lambda=(m/n)\left[\alpha-\frac{3\beta}{4m}\right]\Big/\left[\alpha-\frac{\beta}{2m}\right]. \tag{4.11}$$

If $\alpha>\beta/2$, and $m\geqq 3/2$, then $\lambda>0$. This implies that there are too many firms

[7]The second-order conditions require that the determinant of the bordered Hessian matrix H of L be positive at the second-best optimum, where

$$\det[H]=(3\beta C/32m^4n)[(\alpha-\beta/(3m))(\alpha-(\beta/m))]/(\alpha-\beta/(2m)).$$

This is positive for $m\geqq 2$.

at the second-best optimum: an increase in fixed costs, which induces firms to exit (or not enter), the city, leads to higher expected utility per worker.

Why is entry excessive under some conditions? There are two partially offsetting externalities associated with entry in this model. One is the conventional productivity externality: as the number of firms rises, the expected quality of the match between job requirements and skills improves for all firms. Thus, adding another firm to the city increases the expected profits of all firms, but the entrant ignores the effects of his decisions on others. Because of this productivity externality, free entry leads to too few firms, ceteris paribus. The second externality arises from spatial competition and the heterogeneity of workers and firms. As the number of firms rises, competition in the labor market becomes more intense, and market areas shrink. As a result, expected employment and expected profits decrease for all firms. This negative externality is also ignored by the entrant. $\lambda > 0$ when the competition externality dominates, implying that free entry leads to too many firms relative to the first-best optimum.

These externalities also affect the optimum number of workers with free entry. To see this, it is useful to write (4.8) in another form. Eq. (4.10) implicitly defines $m^S(n)$, the zero profit number of firms for n workers, where

$$\frac{\mathrm{d}m^S}{\mathrm{d}n} = -\left.\frac{\partial P/\partial n}{\partial P/\partial m}\right|_{P(n,m)=0} = (m^S/n)\left[\alpha - \frac{\beta}{4m^S}\right]\Big/\left[\alpha - \frac{\beta}{2m^S}\right], \tag{4.12}$$

and $\mathrm{d}m^S/\mathrm{d}n > 0$ for $\alpha > \beta/2$ and $m \geqq 1$. Combining (4.11) and (4.12), λ may be written:

$$\lambda = \frac{\mathrm{d}m^S}{\mathrm{d}n} \cdot \left[\left(\alpha - \frac{3\beta}{4m^S}\right)\Big/\left(\alpha - \frac{\beta}{4m^S}\right)\right]. \tag{4.13}$$

Substituting (4.13) into (4.8) and rearranging terms yields the following expression for the optimum number of workers in a second-best city:

$$(m^S/n^2)C = (t/3)(n\phi)^{-1/2} - (\mathrm{d}m^S/\mathrm{d}n)[\beta/(4(m^S)^2) - C/n]. \tag{4.14}$$

The change in expected per capita utility caused by adding another worker at the second-best optimum has three components. The term on the left in (4.14) is the marginal social benefit of another worker, the decrease in fixed costs per worker created by spreading costs over a larger population. The terms on the right in (4.14) are the marginal social costs of adding another worker. The first term is marginal aggregate transport cost per worker. The second term is the marginal welfare cost of entry. This is the product of the

number of firms who enter in response to a small increase in population, dm^S/dn, and the marginal expected utility of a firm at the second-best optimum, $\beta/(4(m^S)^2)-C/n$. If $\lambda>0$, then $\beta/(4(m^S)^2)-C/n<0$ from (4.9). Comparing (4.14) with (4.4) shows that the marginal social cost of a worker at the second-best optimum exceeds the marginal social cost of a worker at the first-best optimum by precisely the welfare cost of entry.

The Henry George theorem does not hold at the second-best optimum. Multiplying both sides of (4.14) by n^2 yields

$$m^S C=(t/3)\phi^{-1/2}n^{3/2}-n^2(dm^S/dn)[\beta/(4(m^S)^2)-C/n]. \tag{4.15}$$

When $\lambda>0$, implying $\beta/(4(m^S)^2)-C/n<0$ from (4.9), aggregate fixed costs exceed aggregate differential land rents. Since there are too many firms, a confiscatory land rent tax will not provide enough revenue to cover the cost of establishing a constrained optimum city.

5. Equilibrium in a game of city formation

Our primary reason for discussing the second-best optimum is that it is an equilibrium of a simple city formation game. In this game, firms and households choose cities to maximize profits and utility; developers form cities whenever they can profit by doing so.

It has frequently been suggested that individual migration incentives could lead to cities that are too large.[8] Suppose that population is divided into a few very large cities even though many middle-sized cities would provide a higher level of utility to a representative household. This situation cannot be improved upon by one household forming a new city if the large cities provide a level of utility greater than that available under autarky. Thus, it is a Nash equilibrium for some specification of a city formation game.

Henderson (1974, 1988) makes the valuable observation that the actions of developers on the land market internalize inefficiencies that arise from migration. He writes that (1988, p. 39):

> '...equilibrium [city] size may be obtained through the direct actions of economic agents, such as competitive land developers. These are entrepreneurs who set up communities or cities in a national economy. If some city sizes are... [inefficient], developers can earn temporary profits by setting up and selling housing in cities of more efficient size... Developers play an entrepreneurial role that facilitates large movements of people so that a new city can form en masse. Their role is identical to that of firm entrepreneurs in a usual heuristic general equilibrium model

[8]This is one of the many points made in Stiglitz (1977).

with determinant firm size, except rather than designing firms they are designing cities.'

However, in Henderson's model neither the possible strategies of the developers nor the source of the agglomeration economy are clearly specified. Below, we develop a city formation game in which competitive land developers can only achieve a second-best optimum.

The key assumption we make is that the equilibrium number of firms in a city is determined by free entry. Unfortunately, one consequence of the matching agglomeration economy discussed earlier is that the number of firms under free entry is excessive. Without the ability to exclude firms from a city, developers cannot attain a first-best optimum. There are several reasons to doubt a developer's ability to control the number of firms. First, a developer may not be able to obtain all the land in an urban area. Second, even if a developer controls all land, he may not be able to restrict the number of firms. Residential land may be converted to commercial use, and existing commercial land may be used more intensively. Third, developers cannot impose taxes to influence firms' location decisions. For these reasons, land developers are not equivalent to local governments. A local government can enact land use controls and levy taxes; developers cannot.

The game of city formation is played by N households, M potential firms and D potential developers. N, M and D are finite. M and D are sufficiently large that none of our solutions are constrained by the number of available firms or developers. For simplicity, we assume that each developer owns one potential city site. Adding a class of landowners changes none of our results. Workers choose cities to maximize expected utility. Firms choose cities to maximize expected profits. Developers form cities and offer to return a fraction of land rents to workers. In so doing, they maximize the land rents that they retain.

The game proceeds in three stages. In the first stage, some of the potential developers form cities by offering contracts to workers specifying city populations, n, and side payments equal to the fraction of land rents to be returned, δ. Denote the number of active developers by K. At this point, developers know the objective functions of workers and firms. In the second stage, households choose from among the contracts offered by the active developers, knowing K, the objective functions of workers and firms, and the vector of (n, δ) offered contracts. In the third stage, firms choose locations from among the cities formed by the active developers. At this stage, the firms know K, the objective functions of workers and firms, the vector of (n, δ) offered contracts, and the populations of the cities.

We choose the concept of subgame perfection to characterize the solution to this game.[9] A Nash equilibrium requires all players to select strategies

[9]The concept is due to Selten (1975). It is detailed in Van Damme (1987).

that are mutual best responses. The restriction of subgame perfection requires that every move in an equilibrium strategy be part of a Nash equilibrium strategy for the subgame beginning at that move. Such a solution is found by backward induction: first, the mutual best responses of players in the last stage are determined; next, the same is done in the penultimate stage, taking the outcome in the last stage as given, and so on. While solving for the subgame perfect Nash equilibrium, we impose the additional restriction that active developers adopt the same strategies in equilibrium.

In the third stage, each firm takes the number of firms and workers in each city as given and enters the city that maximizes its expected profit. Equilibrium occurs when no firm has an incentive to change its choice. With free entry, the number of firms in a city with population n is given by $m^{\mathrm{S}}(n)$, the zero profit number of firms for n workers.

In the second stage, households choose cities to maximize utility. Each household makes its choice taking the choices of other households and the responses of firms as given. A developer offering contract (n, δ) will generate a utility level $V^{\mathrm{E}}(n, \delta; m)$, where[10]

$$V^{\mathrm{E}}(n, \delta; m) = \alpha - \beta/4m + (\delta t/3)(n/\phi)^{1/2} - (m/n)C - t(n/\phi)^{1/2}. \tag{5.1}$$

Since the number of firms is determined by free entry, we may impose zero profits from (3.4), and rewrite (5.1) as

$$V^{\mathrm{E}}(n, \delta; m^{\mathrm{S}}(n)) = (m^{\mathrm{S}}/n)C + (\delta t/3)(n/\phi)^{1/2} - t(n/\phi)^{1/2}. \tag{5.2}$$

Equilibrium occurs when no household has an incentive to alter its choice. Contracts are filled in descending order according to the utility they afford until the entire population is accommodated.[11]

In stage one, developers form cities and offer (n, δ) contracts, taking the offers of other developers as given. By symmetry, all developers must offer the same contracts. This implies that all households achieve the same level of utility in equilibrium. Suppose a developer assumes that his rivals offer (n_0, δ_0) contracts that give households utility $V^{\mathrm{E}}(n_0, \delta_0; m^{\mathrm{S}}(n_0))$. To attract workers, the developer must offer an (n, δ) contract satisfying

$$V^{\mathrm{E}}(n, \delta; m^{\mathrm{S}}(n)) = V^{\mathrm{E}}(n_0, \delta_0; m^{\mathrm{S}}(n_0)). \tag{5.3}$$

[10]Eq. (5.1) is derived from (2.2) by substituting for R from (2.6), $\mathrm{E}[w]$ from (3.5), $\mathrm{E}[S]$ from (3.6), and $r(z) + tz = t(n/\phi)^{1/2}$.

[11]If the capacity of the contracts is less than N, some workers are forced to accept the autarky utility level, which is normalized to zero. Competition among developers precludes this outcome in equilibrium.

The developer chooses n and δ to maximize retained land rents subject to (5.3). The Lagrangean for this problem is

$$L=(1-\delta)(t/3)\phi^{-1/2}n^{3/2}$$
$$+\xi[(m^{\mathrm{S}}/n)C+(\delta t/3)(n/\phi)^{1/2}-t(n/\phi)^{1/2}-V^{\mathrm{E}}(n_0,\delta_0;m^{\mathrm{S}}(n_0))]. \quad (5.4)$$

The first-order conditions, in addition to the constraint, are:

$$\partial L/\partial\delta=-(t/3)\phi^{-1/2}n^{3/2}+\xi(t/3)(n/\phi)^{1/2}=0, \quad (5.5)$$

$$\partial L/\partial n=(1-\delta)(t/2)(n/\phi)^{1/2}$$
$$+\xi[((1/n)(\mathrm{d}m^{\mathrm{S}}/\mathrm{d}n)-m^{\mathrm{S}}/n^2)C+((\delta-3)t/6)(n\phi)^{-1/2}]=0. \quad (5.6)$$

From (5.5), $\xi=n$. Hence the developer's best response contract is characterized by:

$$(1-\delta)(t/2)(n/\phi)^{1/2}+((\mathrm{d}m^{\mathrm{S}}/\mathrm{d}n)-(m^{\mathrm{S}}/n))C+((\delta-3)t/6)(n/\phi)^{1/2}=0, \quad (5.7)$$

$$(m^{\mathrm{S}}/n)C+(\delta t/3)(n/\phi)^{1/2}-t(n/\phi)^{1/2}-V^{\mathrm{E}}(n_0,\delta_0;m^{\mathrm{S}}(n_0))=0. \quad (5.8)$$

Eqs. (5.7) and (5.8) implicitly define the best response offered contract (n,δ) as functions of n_0 and δ_0: $(n^*(n_0,\delta_0),\delta^*(n_0,\delta_0))$.

There are two equilibrium conditions in this stage. First, the best responses of active developers must be mutually consistent. By symmetry, this requires that $(n_0,\delta_0)=(n^*(n_0,\delta_0),\delta^*(n_0,\delta_0))$. Second, competition between active and potential developers ensures that all active developers expect zero profits, which implies $\delta=1$. With $\delta=1$, (5.7) reduces to

$$((\mathrm{d}m^{\mathrm{S}}/\mathrm{d}n)-(m^{\mathrm{S}}/n))C-(t/3)(n/\phi)^{1/2}=0. \quad (5.9)$$

Eq. (5.9) is equivalent to (4.14), the condition that characterizes the second-best optimum. To see this, it is convenient to work with the following description of the second-best optimum:

$$\max_n\,(m^{\mathrm{S}}/n)C-(2t/3)(n/\phi)^{1/2}. \quad (5.10)$$

Eq. (5.10) is derived by substituting $P(n,m)=0$, from (3.4) into (3.7). Eq. (5.9) is also the first-order condition for this problem – the equilibrium and second-best optimum are the same. Thus, the solution to (5.9) is n^{S}, the second-best optimum city size. From the incentive compatibility constraint

and the uniqueness of the second-best optimum, $V^{E}(n^{S}, 1; m^{S}(n^{S})) = V^{E}(n_0, \delta_0; m^{S}(n_0))$ implies $(n_0, \delta_0) = (n^{S}, 1)$. Thus, $(n^{S}, 1)$ satisfies the mutual best response requirement $(n^{S}, 1) = (n^{*}(n^{S}, 1), \delta^{*}(n^{S}, 1))$, and is a subgame perfect Nash equilibrium of the development game.

Henderson's (1974) observation that land developers correct inefficiencies that arise from migration is partially true in our model. Developers do achieve the best possible allocation subject to the constraint that firms earn zero profits. They cannot achieve a first-best optimum. The market failure that prevents this is the tendency for an excessive number of firms to locate in any given city. Of course, this market failure is a direct consequence of the matching agglomeration economy we study. It may not arise in simpler models where the microfoundations of agglomeration are not explicit.

6. Equilibrium and optimum city sizes

With an explicit agglomeration economy in the labor market, equilibrium city sizes are not optimal. We have shown that there are externalities associated with firm location which make free entry equilibria inefficient, and have argued that since profit maximizing land developers cannot control the number of firms directly, they cannot attain efficient city sizes.

Equilibrium and optimum city sizes are illustrated in fig. 1. The lower half of this figure displays two functions $m^{F}(n)$ and $m^{S}(n)$. $m^{F}(n)$, derived by solving (4.5) for m, describes the optimum number of firms for a given number of workers:

$$m^{F}(n) = (1/2)(\beta/C)^{1/2} n^{1/2}. \tag{6.1}$$

$m^{S}(n)$, derived by solving the zero profit condition (5.4) for m, describes the equilibrium number of firms for a given number of workers:[12]

$$m^{S}(n) = (\alpha/4C)[n + n^{1/2}(n - 2\beta C/\alpha^{2})^{1/2}]. \tag{6.2}$$

Notice that these functions intersect at $n = 9\beta C/(4\alpha^{2})$, the population at which firms earn zero profits at the first-best optimum, and that $m^{S}(n) > m^{F}(n)$ for all $n > 9\beta C/(4\alpha^{2})$. Profits are positive in the region above $m^{S}(n)$. The upper half of fig. 1 illustrates the determination of the number of workers in a city. At the first-best optimum, aggregate fixed costs (AFC) equal aggregate differential land rents (DLR). Using (4.4) and (6.1), this may be written

$$(1/2)(\beta C)^{1/2} n^{1/2} = (t/3)\phi^{-1/2} n^{3/2}. \tag{6.3}$$

[12] $m^{S}(n)$ is the larger root of the quadratic $P(n, m) = 0$.

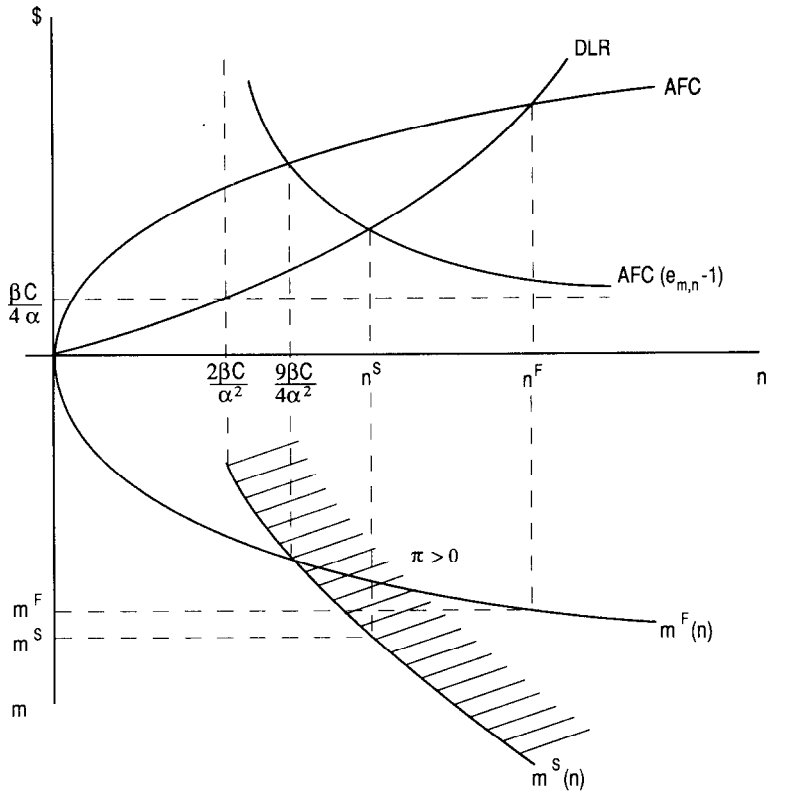

Fig. 1. Optimum and equilibrium city sizes.

Eq. (6.3) is satisfied at the intersection of the curves labelled *AFC* and *DLR* in fig. 1. Notice that *DLR* pivots upward out of the origin in fig. 1. as transport costs increase – for a given population, aggregate differential land rises with transport costs. The optimum numbers of workers and firms are denoted n^F and m^F, respectively. The optimum number of cities is thus $K^F = N/n^F$.

As noted in section 4, aggregate fixed costs do not equal aggregate differential land rents at the second-best optimum. Using (4.14) and (6.2), it can be shown that the equilibrium number of workers in a city satisfies

$$\frac{(\beta C/4\alpha)n^{1/2}}{(n-(2\beta C/\alpha^2))^{1/2}} = (t/3)\phi^{-1/2}n^{3/2}. \tag{6.4}$$

The right side of (4.6) equals aggregate differential land rent. The left side is

related to aggregate fixed costs at the second-best optimum. From (5.8) it equals $m^S C[(dm^S/dn)(n/m^S)-1]$, where the elasticity term accounts for the welfare cost of entry and characterizes the failure of the Henry George theorem. This curve is labelled $AFC[e_{m,n}-1]$ in fig. 1; (6.4) is satisfied at the intersection of $AFC[e_{m,n}-1]$ and DLR. The equilibrium numbers of workers and firms are denoted n^S and m^S, respectively. The equilibrium number of cities is $K^S = N/n^S$.

Before comparing equilibrium and optimum city sizes, it is useful to place some restrictions on the parameters of the model. In particular, we impose assumptions to ensure that expected utility per worker is positive at the optimum. From (4.3) and (B.1) and (B.2) in Appendix B, expected utility at the optimum is

$$V(n^F, m^F) = \alpha - 2(2t/3)^{1/2}(\beta C/\phi)^{1/4}. \tag{6.5}$$

Hence, $t < t^* \equiv (3\alpha^2/8)(\phi/\beta C)^{1/2}$ implies $V(n^F, m^F) > 0$. t^* is the level of transport costs (relative to the other parameters) at which expected utility per worker is zero at the optimum. If transport costs exceed t^*, the model is economically uninteresting. In the remainder of the paper, we express transport costs as a fraction of this critical level: we set $t = \tau t^*$, where $\tau \in (0, 1)$.

Fig. 1 shows that there is some level of transport costs at which optimum and equilibrium city sizes are equal. This is the level of transport costs at which DLR passes through the intersection of AFC and $AFC(e_{m,n}-1)$, or at which $n^F = 9\beta C/(4\alpha^2)$. From (B.1) in Appendix B, this critical level of transport costs is $t = (2\alpha^2/3)(\phi/\beta C)^{1/2} > t^*$. Thus, if we restrict our attention to cases where expected utility is positive at the optimum, DLR must intersect $AFC(e_{m,n}-1)$ and hence AFC to the right of $9\beta C/(4\alpha^2)$. This implies that the equilibrium city contains too many firms *for a given number of workers* ($m^S(n) > m^F(n)$ in this region), and that the equilibrium city contains too few workers. These characteristics arise from the externalities associated with firm location in our model. Since the (negative) competition externality dominates the (positive) productivity externality, free entry leads to an excessive number of firms, and increases the marginal social cost of adding a worker to the city. This in turn reduces the equilibrium population relative to the first-best optimum.

Although firm size is larger at the optimum, since population at the optimum is also greater, the number of firms may be larger at the optimum mutatis mutandis. To address this question, optimum and equilibrium city sizes are simulated for different parameter values in table 1. These simulations assume that the equilibrium city contains 277,400 workers earning an annual wage of \$15,400 from firms containing 13 employees. These assumptions are consistent with 1982 averages for the U.S. calculated from data

Table 1

Simulations.[a]

τ	Column 1: n^F/n^S	Column 2: m^F/m^S	Column 3: $\frac{(1/n^S)ATC}{y^S}$	Column 4: w^F/w^S	Column 5: v^F/v^S	Column 6: (m^SC/DLR)
0.01	11.47	0.58	0.03	0.99	1.88	66.74
0.03	7.78	0.69	0.06	0.99	1.82	31.24
0.05	6.45	0.75	0.09	0.99	1.78	21.77
0.07	5.68	0.79	0.11	0.99	1.76	17.07
0.09	5.15	0.82	0.13	0.99	1.74	14.19
0.11	4.75	0.85	0.15	0.99	1.72	12.22
0.13	4.44	0.87	0.17	0.99	1.70	10.77
0.15	4.18	0.89	0.19	0.99	1.70	9.66
0.17	3.97	0.90	0.20	0.99	1.68	8.76
0.19	3.78	0.91	0.22	0.99	1.67	8.03
0.21	3.62	0.93	0.24	0.99	1.66	7.42
0.23	3.48	0.94	0.25	0.99	1.65	6.90
0.25	3.34	0.95	0.27	0.99	1.64	6.44
0.27	3.22	0.96	0.28	0.99	1.64	6.05
0.29	3.12	0.96	0.30	0.99	1.63	5.70
0.31	3.02	0.97	0.31	0.99	1.63	5.40
0.33	2.93	0.98	0.33	1.00	1.62	5.12
0.35	2.84	0.98	0.34	1.00	1.62	4.87
0.37	2.76	0.99	0.35	1.00	1.62	4.64
0.39	2.69	0.99	0.37	1.00	1.62	4.44
0.41	2.62	1.00	0.38	1.00	1.62	4.25
0.43	2.56	1.00	0.39	1.00	1.62	4.10
0.45	2.50	1.00	0.41	1.00	1.62	3.91
0.47	2.44	1.01	0.42	1.00	1.62	3.77
0.49	2.38	1.01	0.43	1.00	1.62	3.63
0.51	2.33	1.02	0.44	1.00	1.63	3.50
0.53	2.28	1.02	0.46	1.00	1.64	3.38
0.55	2.24	1.02	0.47	1.00	1.64	3.27
0.57	2.19	1.03	0.48	1.01	1.65	3.16
0.59	2.15	1.03	0.49	1.01	1.66	3.06
0.61	2.11	1.03	0.50	1.01	1.68	2.97
0.63	2.07	1.03	0.51	1.01	1.70	2.88
0.65	2.03	1.03	0.53	1.01	1.72	2.80
0.67	1.99	1.03	0.54	1.01	1.74	2.72
0.69	1.96	1.04	0.55	1.01	1.77	2.64
0.71	1.92	1.04	0.56	1.01	1.81	2.57
0.73	1.89	1.04	0.57	1.01	1.85	2.50
0.75	1.86	1.04	0.58	1.01	1.91	2.44
0.77	1.83	1.04	0.59	1.01	1.99	2.38
0.79	1.80	1.04	0.60	1.01	2.09	2.32

[a]These simulations assume that the equilibrium city contains 277,400 workers earning an average wage of $15,400 from firms with 13 employees.

contained in the 1987 *Statistical Abstract of the United States.* Values of the unobservable parameters in the model and the characteristics of the equilibrium and optimum solutions can be inferred from these assumptions and a value of τ.

The first column in table 1 verifies that the equilibrium city contains too few workers for all τ. The second column shows that the equilibrium city contains too many firms when τ is small (transport costs are low relative to the other parameters), and too few firms when τ is large. What is a reasonable value for τ? The third column gives the ratio of aggregate transport costs per worker to income in equilibrium as a function of τ. Data from the national income accounts suggest that this ratio is around 13%. Surveys of consumer expenditure yield somewhat higher values; the U.S. Bureau of Labor Statistics *Consumer Expenditure Survey 1985* suggests that transportation expenditures constitute roughly 18% of average income. This implies that τ lies between 0.09 and 0.15, and in this range the number of firms is too large in equilibrium.

The fourth and fifth columns in table 1 show that although wages are slightly higher in the equilibrium city for small τ, reflecting the greater number of firms, expected utility per worker is higher in the optimum city. The sixth column confirms that the Henry George theorem fails in equilibrium: aggregate fixed costs exceed differential land rents for all τ.

7. Summary

This paper examines the implications of heterogeneity and imperfect information for the costs and benefits of urban size. We derive an agglomeration economy in the labor market from a matching process between workers and firms, and show that this agglomeration economy has the characteristics of a local public good. We illustrate two externalities associated with firm location: a positive productivity externality (which leads to too few firms under free entry) and a negative competition externality (which leads to too many firms under free entry). The competition externality dominates in our model. As a result, free entry leads to too many firms for a given number of workers. This increases the marginal social cost of a worker, reducing population relative to the first-best optimum.

We assume that residential land developers attempt to profit by correcting inefficiencies in cities. However, we also assume that they cannot control the number of firms in a city directly. They can only offer incentive compatible contracts to workers to induce them to migrate to newly formed cities. The number of firms is then determined by free entry. Thus, equilibrium cities contain the zero profit number of firms. However, because of the externalities discussed above, the zero profit number of firms is not efficient. Unless

developers can control the number of firms in a city, the attainment of efficient city sizes requires government intervention.

Appendix A: The job matching problem

This appendix contains technical derivations of expected employment per firm and the expected distance between workers and firms.

$|x-y|$ is a random variable representing the distance between the skills of a worker and the job requirement of the firm at x. For $0<d<(1/2)$, there are two values of y on the unit circle satisfying $|x-y|=d$. Recalling that y is uniformly distributed on the unit circle, this implies that the probability density of $|x-y|$ is given by

$$f(d) \equiv \Pr\{|x-y|=d\}=2, \quad 0<d<(1/2). \tag{A.1}$$

The probability that a worker's skill lies in the firm's market area is

$$\Pr\{|x-y|<1/(2m)\}=2\int_0^{1/(2m)} \mathrm{d}\mu = 1/m. \tag{A.2}$$

To an individual firm, the event of employing a particular worker is a Bernoulli random variable: the worker is either in the firm's market area (the successful outcome) or not (the unsuccessful outcome). From (A.2), the probability of success, in this case, employment, equals the length of the firm's market area, $1/m$. This implies that the number of workers in the firm's market area (the number of successes), $\Omega(x)$, is a binomial random variable with parameters n and $1/m$, and that expected employment equals

$$\mathrm{E}[\Omega]=n/m. \tag{A.3}$$

For $0<d<(1/2)$, the probability of $|x-y|=d$ conditioned on $y\in(x-1/(2m), x+1/(2m))$ is

$$\Pr\{|x-y|=d: y\in(x-1/(2m), x+1/(2m))\}$$

$$=\frac{\Pr\{|x-y|=d, y\in(x-1/(2m), x+1/(2m))\}}{\Pr\{y\in(x-1/(2m), x+1/(2m))\}}. \tag{A.4}$$

Since y is uniformly distributed on the unit circle, the numerator on the right of (A.4) equals 2 for $d<1/(2m)$ and zero otherwise. The denominator on the right of (A.4) equals $1/m$ from (A.2). Hence, the conditional probability density of $|x-y|$ given that $y\in(x-1/(2m), x+1/(2m))$ equals

$$g(|x-y|: y \in (x-1/(2m), x+1/(2m))) = \begin{cases} 2m & d < 1/(2m) \\ 0 & \text{otherwise} \end{cases}. \tag{A.5}$$

Finally, from (A.5), the expected value of $|x-y|$ conditional on $y \in (x-1/(2m), x+1/(2m))$ is

$$\mathrm{E}[|x-y|: y \in (x-1/(2m), x+1/(2m))] = 2m \int_0^{1/(2m)} \sigma \, \mathrm{d}\sigma = 1/(4m). \tag{A.6}$$

Appendix B: Explicit solutions for city sizes

This appendix derives explicit solutions for the numbers of workers and firms in optimum and equilibrium cities.

Explicit solutions for optimum city size are straightforward. From (6.3), the optimum number of workers in a city is

$$n^{\mathrm{F}} = (3/2t)(\phi\beta C)^{1/2}. \tag{B.1}$$

The optimum number of firms is then given by (6.1):

$$m^{\mathrm{F}} = m^{\mathrm{F}}(n^{\mathrm{F}}) = (1/2)(3/2t)\phi^{1/4}\beta^{3/4}C^{-1/4}. \tag{B.2}$$

The equilibrium number of workers in a city, n^{S}, solves the cubic equation implied by (6.4). This equation is

$$n^3 - (2\beta C/\alpha^2)n^2 - \phi(3\beta C/4\alpha t)^2 = 0. \tag{B.3}$$

Setting $t = \tau t^*$, where $t^* = (3\alpha^2/8)(\phi/\beta C)^{1/2}$, as discussed in the text, the solution to (B.3) is

$$n^{\mathrm{S}} = (\beta C/\alpha^2)J(\tau), \tag{B.4}$$

where

$$J(\tau) = (2/\tau^2)^{1/3}A(4\tau^2/27) + 2/3 \tag{B.5}$$

and

$$A(z) = [z+1+(2z+1)^{1/2}]^{1/3} + [z+1-(2z+1)^{1/2}]^{1/3}. \tag{B.6}$$

An expression for the equilibrium number of firms can be derived from (B.4)–(B.6) and (6.2):

$$m^{\mathrm{S}} = m^{\mathrm{S}}(n^{\mathrm{S}}) = (\beta/4\alpha)B[J(\tau)], \tag{B.7}$$

where

$$B[J(\tau)] = J(\tau) + J(\tau)^{1/2}[J(\tau) - 2]^{1/2}. \tag{B.8}$$

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